Electronic Circuits

Electronics explained in one volume, using both theoretical and practical applications.

- New chapter on Raspberry Pi
- Companion website contains free electronic tools to aid learning for students and a question bank for lecturers
- Practical investigations and questions within each chapter help reinforce learning

Mike Tooley provides all the information required to get to grips with the fundamentals of electronics, detailing the underpinning knowledge necessary to appreciate the operation of a wide range of electronic circuits, including amplifiers, logic circuits, power supplies and oscillators. The fourth edition now offers an even more extensive range of topics, with extended coverage of practical areas such as Raspberry Pi.

The book’s content is matched to the latest pre-degree level courses (from Level 2 up to, and including, Foundation Degree and HND), making this an invaluable reference text for all study levels, and its broad coverage is combined with practical case studies based in real-world engineering contexts. In addition, each chapter includes a practical investigation designed to reinforce learning and provide a basis for further practical work.

A new companion website at www.key2electronics.com offers the reader a set of spreadsheet design tools that can be used to simplify circuit calculations, as well as circuit models and templates that will enable virtual simulation of circuits in the book. These are accompanied by online self-test multiple choice questions for each chapter with automatic marking, to enable students to continually monitor their own progress and understanding. A bank of online questions for lecturers to set as assignments is also available.

Mike Tooley has over 30 years’ experience of teaching electrical principles, electronics and avionics to engineers and technicians, previously as Head of Department of Engineering and Vice Principal at Brooklands College in Surrey, UK, and currently works as a consultant and freelance technical author.
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Fundamentals and applications

Fourth edition

Mike Tooley
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*Note that there is an additional chapter and extra resources on the companion website for this title.*  
Visit [www.key2electronics.com](http://www.key2electronics.com) for more information.
Preface

This is the book that I wish I had when I first started exploring electronics over half a century ago. In those days, transistors were only just making their debut and integrated circuits were completely unknown. Of course, since then much has changed but, despite all of the changes, the world of electronics remains a fascinating one. And, unlike most other advanced technological disciplines, electronics is still something that you can ‘do’ at home with limited resources and with a minimal outlay. A soldering iron, a multi-meter and a handful of components are all you need to get started. Except, of course, for some ideas to get you started – and that’s exactly where this book comes in!

The book has been designed to help you understand how electronic circuits work. It will provide you with the basic underpinning knowledge necessary to appreciate the operation of a wide range of electronic circuits, including amplifiers, logic circuits, power supplies and oscillators.

The book is ideal for people who are studying electronics for the first time at any level, including a wide range of school and college courses. It is equally well suited to those who may be returning to study or who may be studying independently as well as those who may need a quick refresher. The book has 19 chapters, each dealing with a particular topic, and ten appendices containing useful information. The approach is topic-based rather than syllabus-based and each major topic looks at a particular application of electronics. The relevant theory is introduced on a progressive basis and delivered in manageable chunks.

In order to give you an appreciation of the solution of simple numerical problems related to the operation of basic circuits, worked examples have been liberally included within the text. In addition, a number of problems can be found at the end of each chapter and solutions are provided at the end of the book. You can use these end-of-chapter problems to check your understanding and also to give you some experience of the ‘short answer’ questions used in most in-course assessments. For good measure, we have included 80 revision problems in Appendix 2.

At the end of the book you will find 22 sample coursework assignments. These should give you plenty of ‘food for thought’ as well as offering you some scope for further experimentation. It is not envisaged that you should complete all of these assignments, and a carefully chosen selection will normally suffice. If you are following a formal course, your teacher or lecturer will explain how these should be tackled and how they can contribute to your course assessment.

While the book assumes no previous knowledge of electronics, you need to be able to manipulate basic formulae and understand some simple trigonometry in order to follow the numerical examples. A study of mathematics to GCSE level (or equivalent) will normally be adequate to satisfy this requirement. However, for those who may need a refresher or have had previous problems with mathematics, Appendix 8 will provide you with the underpinning mathematical knowledge required.

In the later chapters of the book, a number of representative circuits (with component values) have been included together with sufficient information to allow you to adapt and modify the circuits for your own use. These circuits can be used to form the basis of your own practical investigations or they can be combined together in more complex circuits.
Preface

This latest edition brings the book up to date with coverage of several important new topics, including the use of digital storage and sound card oscilloscopes, HDL/VHDL modelling of large-scale logic systems and a completely new chapter devoted to the Raspberry Pi.

Finally, you can learn a great deal from building, testing and modifying simple circuits. To do this you will need access to a few basic tools and some minimal testing equipment. Your first purchase should be a simple multi-range meter, either digital or analogue. This instrument will allow you to measure the voltages and currents present so that you can compare them with the predicted values. If you are attending a formal course of instruction and have access to an electronics laboratory, do make full use of it!
A note for teachers and lecturers

The book is ideal for students following formal courses (e.g. GCSE, AS-, A-level, BTEC, City & Guilds, etc.) in schools, sixth-form colleges and further/higher education colleges. It is equally well suited for use as a text that can support distance or flexible learning and for those who may need a ‘refresher’ before studying electronics at a higher level.

While the book assumes little previous knowledge, students need to be able to manipulate basic formulae and understand some simple trigonometry to follow the numerical examples. A study of mathematics to GCSE level (or beyond) will normally be adequate to satisfy this requirement. However, an appendix has been added specifically to support students who may have difficulty with mathematics. Students will require a scientific calculator in order to tackle the end-of-chapter problems as well as the revision problems that appear at the end of the book.

We have also included 22 sample coursework assignments. These are open-ended and can be modified or extended to suit the requirements of the particular awarding body. The assignments have been divided into those that are broadly at Level 2 and those that are at Level 3. In order to give reasonable coverage of the subject, students should normally be expected to complete four or five of these assignments.

Teachers can differentiate students’ work by mixing assignments from the two levels. In order to challenge students, minimal information should be given to students at the start of each assignment. The aim should be to give students ‘food for thought’ and encourage them to develop their own solutions and interpretation of the topic.

Where this text is to be used to support formal teaching it is suggested that the chapters should be followed broadly in the order that they appear, with the notable exception of Chapter 13. Topics from this chapter should be introduced at an early stage in order to support formal lab work. Assuming a notional delivery time of 4.5 hours per week, the material contained in this book (together with supporting laboratory exercises and assignments) will require approximately two academic terms (i.e. 24 weeks) to deliver, in which the total of 90 hours of study time should be divided equally into theory (supported by problem solving) and practical (laboratory and assignment work). The recommended four or five assignments will require about 25–30 hours of student work to complete.

When developing a teaching programme it is, of course, essential to check that you fully comply with the requirements of the awarding body concerning assessment and that the syllabus coverage is adequate.
A word about safety

When working on electronic circuits, personal safety (both yours and that of those around you) should be paramount in everything you do. Hazards can exist within many circuits – even those that, on the face of it, may appear to be totally safe. Inadvertent misconnection of a supply, incorrect earthing, reverse connection of a high-value electrolytic capacitor and incorrect component substitution can all result in serious hazards to personal safety as a consequence of fire, explosion or the generation of toxic fumes.

Potential hazards can usually be easily recognized and it is well worth making yourself familiar with them, but perhaps the most important point to make is that electricity acts very quickly and you should always think carefully before working on circuits where mains or high voltages (i.e. those over 50V or so) are present. Failure to observe this simple precaution can result in the very real risk of electric shock.

Voltages in many items of electronic equipment, including all items which derive their power from the a.c. mains supply, are at a level which can cause sufficient current flow in the body to disrupt normal operation of the heart. The threshold will be even lower for anyone with a defective heart. Bodily contact with mains or high-voltage circuits can thus be lethal. The most critical path for electric current within the body (i.e. the one that is most likely to stop the heart) is that which exists from one hand to the other. The hand-to-foot path is also dangerous, but somewhat less so than the hand-to-hand path.

So, before you start to work on an item of electronic equipment, it is essential not only to switch off, but to disconnect the equipment at the mains by removing the mains plug. If you have to make measurements or carry out adjustments on an item of working (or ‘live’) equipment, a useful precaution is that of using one hand only to perform the adjustment or to make the measurement. Your ‘spare’ hand should be placed safely away from contact with anything metal (including the chassis of the equipment which may, or may not, be earthed).

The severity of electric shock depends upon several factors, including the magnitude of the current, whether it is alternating or direct current, and its precise path through the body. The magnitude of the current depends upon the voltage which is applied and the resistance of the body. The electrical energy developed in the body will depend upon the time for which the current flows. The duration of contact is also crucial in determining the eventual physiological effects of the shock. As a rough guide, and assuming that the voltage applied is from the 250V, 50Hz a.c. mains supply, the following effects are typical:

<table>
<thead>
<tr>
<th>Current</th>
<th>Physiological effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 1 mA</td>
<td>Not usually noticeable</td>
</tr>
<tr>
<td>1 mA to 2 mA</td>
<td>Threshold of perception (a slight tingle may be felt)</td>
</tr>
<tr>
<td>2 mA to 4 mA</td>
<td>Mild shock (effects of current flow are felt)</td>
</tr>
<tr>
<td>4 mA to 10 mA</td>
<td>Serious shock (shock is felt as pain)</td>
</tr>
<tr>
<td>10 mA to 20 mA</td>
<td>Motor nerve paralysis may occur (unable to let go)</td>
</tr>
<tr>
<td>20 mA to 50 mA</td>
<td>Respiratory control inhibited (breathing may stop)</td>
</tr>
<tr>
<td>More than 50 mA</td>
<td>Ventricular fibrillation of heart muscle (heart failure)</td>
</tr>
</tbody>
</table>
A word about safety

It is important to note that the figures are quoted as a guide – there have been cases of lethal shocks resulting from contact with much lower voltages and at relatively small values of current. The upshot of all this is simply that any potential in excess of 50V should be considered dangerous. Lesser potentials may, under unusual circumstances, also be dangerous. As such, it is wise to get into the habit of treating all electrical and electronic circuits with great care.

Mike Tooley
August 2014
CHAPTER 1

Electrical fundamentals

Chapter summary

This chapter has been designed to provide you with the background knowledge required to help you understand the concepts introduced in the later chapters. If you have studied electrical science, electrical principles or electronics beyond school level then you will already be familiar with many of these concepts. If, on the other hand, you are returning to study or are a newcomer to electronics or electrical technology this chapter will help you get up to speed.
1 Electrical fundamentals

Fundamental units

You will already know that the units that we now use to describe such things as length, mass and time are standardized within the International System of Units. This SI system is based upon the seven fundamental units (see Table 1.1).

Derived units

All other units are derived from these seven fundamental units. These derived units generally have their own names and those commonly encountered in electrical circuits are summarized in Table 1.2 together with the corresponding physical quantities.

Example 1.1

The unit of flux density (the Tesla) is defined as the magnetic flux per unit area. Express this in terms of the fundamental units.

Solution

The SI unit of flux is the Weber (Wb). Area is directly proportional to length squared and, expressed in terms of the fundamental SI units, this is square metres (m²). Dividing the flux (Wb) by the area (m²) gives Wb/m² or Wb m⁻². Hence, in terms of the fundamental SI units, the Tesla is expressed in Wb m⁻².

Table 1.1 SI units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Length</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>candela</td>
<td>cd</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Temperature</td>
<td>Kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Matter</td>
<td>mol</td>
<td>mol</td>
</tr>
</tbody>
</table>

(Note that 0 K is equal to −273°C and an interval of 1 K is the same as an interval of 1°C.)

Table 1.2 Electrical quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Derived unit</th>
<th>Abbreviation</th>
<th>Equivalent (in terms of fundamental units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>Farad</td>
<td>F</td>
<td>A s V⁻¹</td>
</tr>
<tr>
<td>Charge</td>
<td>Coulomb</td>
<td>C</td>
<td>A s</td>
</tr>
<tr>
<td>Energy</td>
<td>Joule</td>
<td>J</td>
<td>N m</td>
</tr>
<tr>
<td>Force</td>
<td>Newton</td>
<td>N</td>
<td>kg m s⁻¹</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hertz</td>
<td>Hz</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>Illuminance</td>
<td>Lux</td>
<td>lx</td>
<td>lm m⁻²</td>
</tr>
<tr>
<td>Inductance</td>
<td>Henry</td>
<td>H</td>
<td>V s A⁻¹</td>
</tr>
<tr>
<td>Luminous flux</td>
<td>Lumen</td>
<td>lm</td>
<td>cd sr</td>
</tr>
<tr>
<td>Magnetic flux</td>
<td>Weber</td>
<td>Wb</td>
<td>V s</td>
</tr>
<tr>
<td>Potential</td>
<td>Volt</td>
<td>V</td>
<td>W A⁻¹</td>
</tr>
<tr>
<td>Power</td>
<td>Watt</td>
<td>W</td>
<td>J s⁻¹</td>
</tr>
<tr>
<td>Resistance</td>
<td>Ohm</td>
<td>Ω</td>
<td>V A⁻¹</td>
</tr>
</tbody>
</table>

Example 1.2

The unit of electrical potential, the volt (V), is defined as the difference in potential between two points in a conductor which, when carrying a current of one amp (A), dissipates a power of one watt (W). Express the volt (V) in terms of joules (J) and coulombs (C).

Solution

In terms of the derived units:

\[
V = \frac{W}{A} = \frac{J}{A s} = \frac{J}{A s} = \frac{J}{C} = JC^{-1}
\]

Hence, one volt is equivalent to one joule per coulomb.
Measuring angles

You might think it strange to be concerned with angles in electrical circuits. The reason is simply that, in analogue and a.c. circuits, signals are based on repetitive waves (often sinusoidal in shape). We can refer to a point on such a wave in one of two basic ways, either in terms of the time from the start of the cycle or in terms of the angle (a cycle starts at 0° and finishes as 360° (see Fig. 1.1)). In practice, it is often more convenient to use angles rather than time; however, the two methods of measurement are interchangeable and it’s important to be able to work in either of these units.

In electrical circuits, angles are measured in either degrees or radians (both of which are strictly dimensionless units). You will doubtless already be familiar with angular measure in degrees where one complete circular revolution is equivalent to an angular change of 360°. The alternative method of measuring angles, the radian, is defined somewhat differently. It is the angle subtended at the centre of a circle by an arc having length which is equal to the radius of the circle (see Fig. 1.2).

You may sometimes find that you need to convert from radians to degrees, and vice versa. A complete circular revolution is equivalent to a rotation of 360° or \(2\pi\) radians (note that \(\pi\) is approximately equal to 3.142). Thus one radian is equivalent to 360/\(2\pi\) degrees (or approximately 57.3°). Try to remember the following rules that will help you to convert angles expressed in degrees to radians and vice versa:

- From degrees to radians, divide by 57.3.
- From radians to degrees, multiply by 57.3.

Example 1.3
Express a quarter of a cycle revolution in terms of:
(a) degrees;
(b) radians.

Solution
(a) There are 360° in one complete cycle (i.e. one full revolution). Hence there are \((360/4)°\) or 90° in one-quarter of a cycle.

Example 1.4
Express an angle of 215° in radians.

Solution
To convert from degrees to radians, divide by 57.3. So 215° is equivalent to 215/57.3 = 3.75 radians.

Example 1.5
Express an angle of 2.5 radians in degrees.

Solution
To convert from radians to degrees, multiply by 57.3. Hence 2.5 radians is equivalent to 2.5 \(\times\) 57.3 = 143.25°.
1 Electrical fundamentals

Electrical units and symbols

Table 1.3 shows the units and symbols that are commonly encountered in electrical circuits. It is important to get to know these units and also be able to recognize their abbreviations and symbols. You will meet all of these units later in this chapter.

Multiples and sub-multiples

Unfortunately, many of the derived units are either too large or too small for convenient everyday use but we can make life a little easier by using a standard range of multiples and sub-multiples (see Table 1.4).

Example 1.6
An indicator lamp requires a current of 0.075 A. Express this in mA.

Solution
You can express the current in mA (rather than in A) by simply moving the decimal point three places to the right. Hence 0.075 A is the same as 75 mA.

Example 1.7
A medium-wave radio transmitter operates on a frequency of 1,495 kHz. Express its frequency in MHz.

Solution
To express the frequency in MHz rather than kHz we need to move the decimal point three places to the left. Hence 1,495 kHz is equivalent to 1.495 MHz.

Example 1.8
Express the value of a 27,000 pF in μF.

Solution
To express the value in μF rather than pF we need to move the decimal point six places to the left. Hence 27,000 pF is equivalent to 0.027 μF (note that we have had to introduce an extra zero before the 2 and after the decimal point).

<table>
<thead>
<tr>
<th>Table 1.3 Electrical units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit</strong></td>
</tr>
<tr>
<td>Ampere</td>
</tr>
<tr>
<td>Coulomb</td>
</tr>
<tr>
<td>Farad</td>
</tr>
<tr>
<td>Henry</td>
</tr>
<tr>
<td>Hertz</td>
</tr>
<tr>
<td>Joule</td>
</tr>
<tr>
<td>Ohm</td>
</tr>
<tr>
<td>Second</td>
</tr>
<tr>
<td>Siemen</td>
</tr>
<tr>
<td>Tesla</td>
</tr>
<tr>
<td>Volt</td>
</tr>
<tr>
<td>Watt</td>
</tr>
<tr>
<td>Weber</td>
</tr>
</tbody>
</table>
Exponent notation

Exponent notation (or scientific notation) is useful when dealing with either very small or very large quantities. It’s well worth getting to grips with this notation as it will allow you to simplify quantities before using them in formulae.

Exponents are based on powers of ten. To express a number in exponent notation the number is split into two parts. The first part is usually a number in the range 0.1 to 100 while the second part is a multiplier expressed as a power of ten.

For example, 251.7 can be expressed as 2.517 × 10^2. It can also be expressed as 0.2517 × 1,000, i.e. 0.2517 × 10^3. In both cases the exponent is the same as the number of noughts in the multiplier (i.e. 2 in the first case and 3 in the second case). To summarize:

\[
251.7 = 2.517 \times 10^2 = 0.2517 \times 10^3
\]

As a further example, 0.01825 can be expressed as 1.825/100, i.e. 1.825 × 10^-2. It can also be expressed as 18.25/1,000, i.e. 18.25 × 10^-3. Again, the exponent is the same as the number of noughts but the minus sign is used to denote a fractional multiplier. To summarize:

\[
0.01825 = 1.825 \times 10^{-2} = 18.25 \times 10^{-3}
\]

Example 1.9
A current of 7.25 mA flows in a circuit. Express this current in amperes using exponent notation.

\[
1 \text{ mA} = 1 \times 10^{-3} \text{ A thus } 7.25 \text{ mA} = 7.25 \times 10^{-3} \text{ A.}
\]

Example 1.10
A voltage of 3.75 × 10^-6 V appears at the input of an amplifier. Express this voltage in (a) V and (b) mV, using exponent notation.

Solution
(a) \(1 \times 10^{-6} \text{ V} = 1 \mu \text{V so } 3.75 \times 10^{-6} \text{ V} = 3.75 \mu \text{V.}\)
(b) There are 1,000 \(\mu \text{V}\) in 1 mV so we must divide the previous result by 1,000 in order to express the voltage in mV. So \(3.75 \mu \text{V} = 0.00375 \text{ mV.}\)

Multiplication and division using exponents

Exponent notation really comes into its own when values have to be multiplied or divided. When multiplying two values expressed using exponents, you simply need to add the exponents. Here’s an example:

\[
(2 \times 10^2) \times (3 \times 10^5) = (2 \times 3) \times 10^{2+5} = 6 \times 10^7
\]

Similarly, when dividing two values which are expressed using exponents, you only need to subtract the exponents. As an example:

\[
(4 \times 10^6) \div (2 \times 10^3) = 4/2 \times 10^{6-3} = 2 \times 10^3
\]

In either case it’s important to remember to specify the units, multiples and sub-multiples in which you are working (e.g. A, kΩ, mV, \(\mu\text{F, etc.}\)).

Example 1.11
A current of 3 mA flows in a resistance of 33 kΩ. Determine the voltage dropped across the resistor.

Solution
Voltage is equal to current multiplied by resistance (see page 7). Thus:

\[
V = I \times R = 3 \text{ mA} \times 33 \text{ k}\Omega
\]

Expressing this using exponent notation gives:

\[
V = (3 \times 10^{-3}) \times (33 \times 10^3) \text{ V}
\]

Separating the exponents gives:

\[
V = 3 \times 33 \times 10^{-3} \times 10^3 \text{ V}
\]

Thus \(V = 99 \times 10^{-3} = 99 \times 10^0 = 99 \times 1 = 99 \text{ V.}\)
1 Electrical fundamentals

Example 1.12
A current of 45 μA flows in a circuit. What charge is transferred in a time interval of 20 ms?

Solution
Charge is equal to current multiplied by time (see the definition of the ampere on page 4). Thus:
\[ Q = I \times t = 45 \, \mu A \times 20 \, ms \]
Expressing this in exponent notation gives:
\[ Q = (45 \times 10^{-6}) \times (20 \times 10^{-3}) \, \text{coulomb} \]
Separating the exponents gives:
\[ Q = 45 \times 20 \times 10^{-6} \times 10^{-3} \, \text{coulomb} \]
Thus \[ Q = 900 \times 10^{-6-3} = 900 \times 10^{-9} = 900 \, \text{nC} \]

Example 1.13
A power of 300 mW is dissipated in a circuit when a voltage of 1,500 V is applied. Determine the current supplied to the circuit.

Solution
Current is equal to power divided by voltage (see page 9). Thus:
\[ I = P / V = 300 \, \text{mW} / 1,500 \, \text{V} \, \text{amperes} \]
Expressing this in exponent notation gives:
\[ I = (300 \times 10^{-3}) / (1.5 \times 10^3) \, \text{A} \]
Separating the exponents gives:
\[ I = (300/1.5) \times (10^{-3}/10^3) \, \text{A} \]
\[ I = 200 \times 10^{-3-3} = 200 \times 10^{-6} = 200 \, \mu A \]

Conductors and insulators

Electric current is the name given to the flow of electrons (or negative charge carriers). Electrons orbit around the nucleus of atoms just as the Earth orbits around the sun (see Fig. 1.3). Electrons are held in one or more shells, constrained to their orbital paths by virtue of a force of attraction towards the nucleus which contains an equal number of protons (positive charge carriers).

Since like charges repel and unlike charges attract, negatively charged electrons are attracted to the positively charged nucleus. A similar principle can be demonstrated by observing the attraction between two permanent magnets; the two north poles of the magnets will repel each other, while a north and south pole will attract. In the same way, the unlike charges of the negative electron and the positive proton experience a force of mutual attraction.

The outer-shell electrons of a conductor can be reasonably easily interchanged between adjacent atoms within the lattice of atoms of which the substance is composed. This makes it possible for the material to conduct electricity. Typical examples of conductors are metals such as copper, silver, iron and aluminium. By contrast, the outer-shell electrons of an insulator are firmly bound to their parent atoms and virtually no interchange of electrons is possible. Typical examples of insulators are plastics, rubber and ceramic materials.

Voltage and resistance

The ability of an energy source (e.g. a battery) to produce a current within a conductor may be expressed in terms of electromotive force (e.m.f.). Whenever an e.m.f. is applied to a circuit a potential difference (p.d.) exists. Both e.m.f. and p.d. are measured in volts (V). In many practical circuits there is only one e.m.f. present (the battery or supply) whereas a p.d. will be developed across each component present in the circuit.

The conventional flow of current in a circuit is from the point of more positive potential to the point of greatest negative potential (note that...
1 Electrical fundamentals

The formula may be arranged to make $V$, $I$ or $R$ the subject, as follows:

$$V = I \times R, \quad I = \frac{V}{R} \text{ and } R = \frac{V}{I}$$

The triangle shown in Fig. 1.5 should help you remember these three important relationships. However, it’s worth noting that, when performing calculations of currents, voltages and resistances in practical circuits it is seldom necessary to work with an accuracy of better than ±1% simply because component tolerances are usually greater than this. Furthermore, in calculations involving Ohm’s Law, it can sometimes be convenient to work in units of kΩ and mA (or MΩ and μA) in which case potential differences will be expressed directly in V.

**Example 1.14**

A 12 Ω resistor is connected to a 6 V battery. What current will flow in the resistor?

**Solution**

Here we must use $I = \frac{V}{R}$ (where $V = 6$ V and $R = 12$ Ω):

$$I = \frac{6}{12} = 0.5 \text{ A (or 500 mA)}$$

Hence a current of 500 mA will flow in the resistor.

**Example 1.15**

A current of 100 mA flows in a 56 Ω resistor. What voltage drop (potential difference) will be developed across the resistor?
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**Solution**

Here we must use \( V = I \times R \) and ensure that we work in units of volts (V), amperes (A) and ohms (Ω).

\[ V = I \times R = 0.1 \, \text{A} \times 56 \, \Omega = 5.6 \, \text{V} \]

(Note that 100 mA is the same as 0.1 A.)

This calculation shows that a p.d. of 5.6 V will be developed across the resistor.

**Example 1.16**

A voltage drop of 15 V appears across a resistor in which a current of 1 mA flows. What is the value of the resistance?

**Solution**

\[ R = \frac{V}{I} = 15 \, \text{V} / 0.001 \, \text{A} = 15,000 \, \Omega = 15 \, \text{kΩ} \]

Note that it is often more convenient to work in units of mA and V, which will produce an answer directly in kΩ, i.e.

\[ R = \frac{V}{I} = 15 \, \text{V} / 1 \, \text{mA} = 15 \, \text{kΩ} \]

**Resistance and resistivity**

The resistance of a metallic conductor is directly proportional to its length and inversely proportional to its area. The resistance is also directly proportional to its resistivity (or specific resistance). Resistivity is defined as the resistance measured between the opposite faces of a cube having sides of 1 cm.

The resistance, \( R \), of a conductor is thus given by the formula:

\[ R = \rho \frac{l}{A} \]

where \( R \) is the resistance (ft), \( \rho \) is the resistivity (Ωm), \( l \) is the length (m) and \( A \) is the area (m²).

Table 1.5 shows the electrical properties of some common metals.

**Example 1.17**

A coil consists of an 8 m length of annealed copper wire having a cross-sectional area of 1 mm². Determine the resistance of the coil.

**Solution**

We will use the formula \( R = \rho l / A \).

The value of \( \rho \) for annealed copper given in Table 1.5 is \( 1.724 \times 10^{-8} \) Ωm. The length of the wire is 4 m while the area is 1 mm² or \( 1 \times 10^{-6} \) m² (note that it is important to be consistent in using units of metres for length and square metres for area).

Hence the resistance of the coil will be given by:

\[ R = \frac{1.724 \times 10^{-8} \times 8}{1 \times 10^{-6}} = 13.724 \times 10^{-2} \]

Thus \( R = 13.792 \times 10^{-2} \) or 0.13792 Ω.

---

### Table 1.5 Properties of some common metals

<table>
<thead>
<tr>
<th>Metal</th>
<th>Resistivity (at 20°C) (Ωm)</th>
<th>Relative conductivity (copper = 1)</th>
<th>Temperature coefficient of resistance (per °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>( 1.626 \times 10^{-8} )</td>
<td>1.06</td>
<td>0.0041</td>
</tr>
<tr>
<td>Copper (annealed)</td>
<td>( 1.724 \times 10^{-8} )</td>
<td>1.00</td>
<td>0.0039</td>
</tr>
<tr>
<td>Copper (hard drawn)</td>
<td>( 1.777 \times 10^{-8} )</td>
<td>0.97</td>
<td>0.0039</td>
</tr>
<tr>
<td>Aluminium</td>
<td>( 2.803 \times 10^{-8} )</td>
<td>0.61</td>
<td>0.0040</td>
</tr>
<tr>
<td>Mild steel</td>
<td>( 1.38 \times 10^{-7} )</td>
<td>0.12</td>
<td>0.0045</td>
</tr>
<tr>
<td>Lead</td>
<td>( 2.14 \times 10^{-7} )</td>
<td>0.08</td>
<td>0.0040</td>
</tr>
<tr>
<td>Nickel</td>
<td>( 8.0 \times 10^{-8} )</td>
<td>0.22</td>
<td>0.0062</td>
</tr>
</tbody>
</table>
Example 1.18
A wire having a resistivity of $1.724 \times 10^{-8} \, \Omega \text{m}$, length 20 m and cross-sectional area 1 mm$^2$ carries a current of 5 A. Determine the voltage drop between the ends of the wire.

Solution
First we must find the resistance of the wire (as in Example 1.17):

$$R = \frac{\rho l}{A} = \frac{1.6 \times 10^{-8} \times 20}{1 \times 10^{-6}} = 32 \times 10^{-2} = 0.32 \, \Omega$$

The voltage drop can now be calculated using Ohm’s Law:

$$V = I \times R = 5A \times 0.32 \, \Omega = 1.6 \, V$$

This calculation shows that a potential of 1.6 V will be dropped between the ends of the wire.

Energy and power
At first you may be a little confused about the difference between energy and power. Put simply, energy is the ability to do work while power is the rate at which work is done. In electrical circuits, energy is supplied by batteries or generators. It may also be stored in components such as capacitors and inductors. Electrical energy is converted into various other forms of energy by components such as resistors (producing heat), loudspeakers (producing sound energy) and light emitting diodes (producing light).

The unit of energy is the joule (J). Power is the rate of use of energy and it is measured in watts (W). A power of 1 W results from energy being used at the rate of 1 J per second. Thus:

$$P = \frac{W}{t}$$

where $P$ is the power in watts (W), $W$ is the energy in joules (J) and $t$ is the time in seconds (s).

The power in a circuit is equivalent to the product of voltage and current. Hence:

$$P = I \times V$$

where $P$ is the power in watts (W), $I$ is the current in amperes (A) and $V$ is the voltage in volts (V).

The formula may be arranged to make $P$, $I$ or $V$ the subject, as follows:

$$P = I \times V, \quad I = P / V \text{ and } V = P / I$$

The triangle shown in Fig. 1.6 should help you remember these relationships.

The relationship, $P = I \times V$, may be combined with that which results from Ohm’s Law ($V = I \times R$) to produce two further relationships. First, substituting for $V$ gives:

$$P = I \times (I \times R) = I^2 R$$

Second, substituting for $I$ gives:

$$P = \frac{V}{R} \times V = V^2 / R$$

Example 1.19
A current of 1.5 A is drawn from a 3 V battery. What power is supplied?

Solution
Here we must use $P = I \times V$ (where $I = 1.5$ A and $V = 3$ V).

$$P = I \times V = 1.5 \, A \times 3 \, V = 4.5 \, W$$

Hence a power of 4.5 W is supplied.

Example 1.20
A voltage drop of 4 V appears across a resistor of 100 $\Omega$. What power is dissipated in the resistor?

Solution
Here we use $P = V^2 / R$ (where $V = 4$ V and $R = 100 \, \Omega$).

$$P = \frac{V^2}{R} = \frac{(4 \, V \times 4 \, V)}{100 \, \Omega} = 0.16 \, W$$

Hence the resistor dissipates a power of 0.16 W (or 160 mW).
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Example 1.21
A current of 20 mA flows in a 1 kΩ resistor. What power is dissipated in the resistor?

Solution
Here we use \( P = I^2 \times R \) but, to make life a little easier, we will work in mA and kΩ (in which case the answer will be in mW).

\[
P = I^2 \times R = (20 \text{ mA} \times 20 \text{ mA}) \times 1 \text{ kΩ} = 400 \text{ mW}
\]

Thus a power of 400 mW is dissipated in the 1 kΩ resistor.

Electric fields
The force exerted on a charged particle is a manifestation of the existence of an electric field. The electric field defines the direction and magnitude of a force on a charged object. The field itself is invisible to the human eye but can be drawn by constructing lines which indicate the motion of a free positive charge within the field; the number of field lines in a particular region is used to indicate the relative strength of the field at the point in question.

Figs 1.7 and 1.8 show the electric fields between charges of the same and opposite polarity while Fig. 1.9 shows the field which exists between two charged parallel plates. You will see more of this particular arrangement when we introduce capacitors in Chapter 2.

Electric field strength
The strength of an electric field (\( E \)) is proportional to the applied potential difference and inversely proportional to the distance between the two conductors. The electric field strength is given by:

\[
E = \frac{V}{d}
\]

where \( E \) is the electric field strength (V/m), \( V \) is the applied potential difference (V) and \( d \) is the distance (m).

Combining the two previous equations gives:

\[
F = \frac{kQ_1Q_2}{4\pi \times 8.854 \times 10^{-12} r^2} \text{ newtons}
\]

Electrostatics
If a conductor has a deficit of electrons, it will exhibit a net positive charge. If, on the other hand, it has a surplus of electrons, it will exhibit a net negative charge. An imbalance in charge can be produced by friction (removing or depositing electrons using materials such as silk and fur, respectively) or induction (by attracting or repelling electrons using a second body which is, respectively, positively or negatively charged).

Force between charges
Coulomb’s Law states that if charged bodies exist at two points, the force of attraction (if the charges are of opposite polarity) or repulsion (if the charges have the same polarity) will be proportional to the product of the magnitude of the charges divided by the square of their distance apart. Thus:

\[
F = \frac{kQ_1Q_2}{r^2}
\]

where \( Q_1 \) and \( Q_2 \) are the charges present at the two points (in coulombs), \( r \) is the distance separating the two points (in metres), \( F \) is the force (in newtons) and \( k \) is a constant depending upon the medium in which the charges exist.

In vacuum or ‘free space’,

\[
k = \frac{1}{4\pi \varepsilon_0}
\]

where \( \varepsilon_0 \) is the permittivity of free space (8.854 \times 10^{-12} \text{ C/Nm}^2).

Figure 1.7 Electric field between two unlike electric charges
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...to have a very high value of resistivity (they must not conduct charge) coupled with an ability to withstand high voltages without breaking down. A more practical arrangement is shown in Fig. 1.10. In this arrangement the ratio of charge, $Q$, to potential difference, $V$, is given by the relationship:

$$\frac{Q}{V} = \epsilon \frac{A}{d}$$

where $A$ is the surface area of the plates (in m$^2$), $d$ is the separation (in m) and $\epsilon$ is a constant for the dielectric material known as the absolute permittivity of the material (sometimes also referred to as the dielectric constant).

The absolute permittivity of a dielectric material is the product of the permittivity of free space ($\epsilon_0$) and the relative permittivity ($\epsilon_r$) of the material. Thus:

$$\epsilon = \epsilon_0 \times \epsilon_r, \quad \text{and} \quad \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d}$$

The dielectric strength of an insulating dielectric is the maximum electric field strength that can safely be applied to it before breakdown (conduction) occurs. Table 1.6 shows values of relative permittivity and dielectric strength for some common dielectric materials.

**Electromagnetism**

When a current flows through a conductor a magnetic field is produced in the vicinity of the conductor. The magnetic field is invisible but
1 Electrical fundamentals

**Table 1.6** Properties of some common insulating dielectric materials

<table>
<thead>
<tr>
<th>Dielectric material</th>
<th>Relative permittivity (free space = 1)</th>
<th>Dielectric strength (kV/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum, or free space</td>
<td>1</td>
<td>∞</td>
</tr>
<tr>
<td>Air</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Polythene</td>
<td>2.3</td>
<td>50</td>
</tr>
<tr>
<td>Paper</td>
<td>2.5 to 3.5</td>
<td>14</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.5</td>
<td>25</td>
</tr>
<tr>
<td>Mica</td>
<td>4 to 7</td>
<td>160</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>4.5</td>
<td>13</td>
</tr>
<tr>
<td>Glass ceramic</td>
<td>5.9</td>
<td>40</td>
</tr>
<tr>
<td>Polyester</td>
<td>3.0 to 3.4</td>
<td>18</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6.5</td>
<td>4</td>
</tr>
<tr>
<td>Titanium dioxide</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>Ceramics</td>
<td>5 to 1,000</td>
<td>2 to 10</td>
</tr>
</tbody>
</table>

its presence can be detected using a compass needle (which will deflect from its normal north–south position). If two current-carrying conductors are placed in the vicinity of one another, the fields will interact with one another and the conductors will experience a force of attraction or repulsion (depending upon the relative direction of the two currents).

**Force between two current-carrying conductors**

The mutual force which exists between two parallel current-carrying conductors will be proportional to the product of the currents in the two conductors and the length of the conductors, but inversely proportional to their separation. Thus:

\[ F = \frac{k l_1 l_2}{d} \]

where \( l_1 \) and \( l_2 \) are the currents in the two conductors (in amperes), \( l \) is the parallel length of the conductors (in metres), \( d \) is the distance separating the two conductors (in metres), \( F \) is the force (in newtons) and \( k \) is a constant depending upon the medium in which the charges exist.

In vacuum or ‘free space’,

\[ k = \frac{\mu_0}{2\pi} \]

where \( \mu_0 \) is a constant known as the permeability of free space (4\(\pi \times 10^{-7} \) or 12.57 \(\times 10^{-7} \) H/m).

Combining the two previous equations gives:

\[ F = \frac{\mu_0 l_1 l_2}{2\pi d} \]

or

\[ F = \frac{4\pi \times 10^{-7} l_1 l_2}{2\pi d} \]

or

\[ F = \frac{2 \times 10^{-7} l_1 l_2}{d} \text{ newtons} \]

**Magnetic fields**

The field surrounding a straight current-carrying conductor is shown in Fig. 1.11. The magnetic field defines the direction of motion of a free north pole within the field. In the case of Fig. 1.11, the lines of flux are concentric and the direction of the field (determined by the direction of current flow) is given by the right-hand rule.

**Magnetic field strength**

The strength of a magnetic field is a measure of the density of the flux at any particular point. In the case of Fig. 1.11, the field strength will be proportional to the applied current and inversely proportional to the perpendicular distance from the conductor. Thus:

\[ B = \frac{k l}{d} \]

where \( B \) is the magnetic flux density (in tesla), \( l \) is the current (in amperes), \( d \) is the distance from the conductor (in metres) and \( k \) is a constant.

Assuming that the medium is vacuum or ‘free space’, the density of the magnetic flux will be given by:
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Solution

Applying the formula $B = \mu_0 \frac{I}{2\pi d}$ gives:

$$B = \frac{12.57 \times 10^{-7} \times 20}{2 \times 3.142 \times 50 \times 10^{-3}} = \frac{251.4 \times 10^{-7}}{314.2 \times 10^{-7}}$$

from which:

$$B = 0.8 \times 10^{-4} \text{ tesla}$$

Thus $B = 80 \times 10^{-6} \text{ T}$ or $B = 80 \mu\text{T}$.

Example 1.24

A flux density of 2.5 mT is developed in free space over an area of 20 cm$^2$. Determine the total flux.

Solution

Re-arranging the formula $B = \Phi / A$ to make $\Phi$ the subject gives $\Phi = B \times A$, thus:

$$\Phi = (2.5 \times 10^{-3}) \times (20 \times 10^{-4}) = 50 \times 10^{-7} \text{ webers}$$

from which $B = 5 \mu\text{Wb}$.

Magnetic circuits

Materials such as iron and steel possess considerably enhanced magnetic properties. Hence they are employed in applications where it is necessary to increase the flux density produced by an electric current. In effect, magnetic materials allow us to channel the electric flux into a ‘magnetic circuit’, as shown in Fig. 1.14.

In the circuit of Fig. 1.14(b) the reluctance of the magnetic core is analogous to the resistance
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![Diagram of Magnetic Field](image1)

**Figure 1.13** The magnetic field surrounding a solenoid coil resembles that of a permanent magnet

present in the electric circuit shown in Fig. 1.14(a). We can make the following comparisons between the two types of circuit (see Table 1.7).

In practice, not all of the magnetic flux produced in a magnetic circuit will be concentrated within the core and some 'leakage flux' will appear in the surrounding free space (as shown in Fig. 1.15). Similarly, if a gap appears within the magnetic circuit, the flux will tend to spread out as shown in Fig. 1.16. This effect is known as ‘fringing’.

**Reluctance and permeability**

The reluctance of a magnetic path is directly proportional to its length and inversely proportional to its area. The reluctance is also inversely proportional to the **absolute permeability** of the magnetic material. Thus:

\[
S = \frac{l}{\mu A}
\]

where \(S\) is the reluctance of the magnetic path, \(l\) is the length of the path (in metres), \(A\) is the cross-sectional area of the path (in square metres) and \(\mu\) is the absolute permeability of the magnetic material.

The absolute permeability of a magnetic material is the product of the permeability of free space (\(\mu_0\)) and the **relative permeability** of the magnetic medium (\(\mu_r\)). Thus

\[
\mu = \mu_0 \times \mu_r \quad \text{and} \quad S = \frac{l}{\mu_0 \mu_r A}
\]
The permeability of a magnetic medium is a measure of its ability to support magnetic flux and it is equal to the ratio of flux density ($B$) to magnetizing force ($H$). Thus:

$$\mu = \frac{B}{H}$$

where $B$ is the flux density (in tesla) and $H$ is the magnetizing force (in amperes/metre). The magnetizing force ($H$) is proportional to the product of the number of turns and current but inversely proportional to the length of the magnetic path.

$$H = \frac{NI}{l}$$

where $H$ is the magnetizing force (in amperes/metre), $N$ is the number of turns, $I$ is the current.

**Table 1.7** Comparison of electric and magnetic circuits

<table>
<thead>
<tr>
<th>Electric circuit</th>
<th>Magnetic circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.14(a)</td>
<td>Figure 1.14(a)</td>
</tr>
<tr>
<td>Electromotive force, e.m.f. $= V$</td>
<td>Magnetomotive force, m.m.f. $= NI$</td>
</tr>
<tr>
<td>Resistance $= R$</td>
<td>Reluctance $= S$</td>
</tr>
<tr>
<td>Current $= I$</td>
<td>Flux $= \Phi$</td>
</tr>
<tr>
<td>e.m.f. = current $\times$ resistance</td>
<td>m.m.f. = flux $\times$ reluctance</td>
</tr>
<tr>
<td>$V = I \times R$</td>
<td>$NI = S \Phi$</td>
</tr>
</tbody>
</table>

**B–H curves**

Fig. 1.17 shows four typical $B–H$ (flux density plotted against permeability) curves for some common magnetic materials. If you look carefully at these curves you will notice that they flatten off due to magnetic saturation and that the slope of the curve (indicating the value of $\mu$ corresponding to a particular value of $H$) falls as the magnetizing force increases. This is important since it dictates the acceptable working range for a particular magnetic material when used in a magnetic circuit.

**Example 1.25**

Estimate the relative permeability of cast steel (see Fig. 1.18) at (a) a flux density of 0.6 T and (b) a flux density of 1.6 T.

**Solution**

From Fig. 1.18, the slope of the graph at any point gives the value of $\mu$ at that point. We can easily find the slope by constructing a tangent at
now be determined by re-arranging \( H = \frac{N I}{l} \) as

\[
I = \frac{H \times l}{N} = \frac{3,500 \times 0.6}{800} = 2.625 \text{ A}
\]

**Circuit diagrams**

Finally, and just in case you haven’t seen them before, we will end this chapter with a brief word about circuit diagrams. We are introducing the topic here because it’s quite important to be able to read and understand simple electronic circuit diagrams before you can make sense of some of the components and circuits that you will meet later.

Circuit diagrams use standard symbols and conventions to represent the components and wiring used in an electronic circuit. Visually, they bear very little relationship to the physical layout of a circuit but, instead, they provide us with a ‘theoretical’ view of the circuit. In this section we show you how to find your way round simple circuit diagrams.

To be able to understand a circuit diagram you first need to be familiar with the symbols that are used to represent the components and devices. A selection of some of the most commonly used symbols are shown later, in Fig. 1.24.

It’s important to be aware that there are a few (thankfully quite small) differences between the symbols used in circuit diagrams of American and European origin.

As a general rule, the input to a circuit should be shown on the left of a circuit diagram and the output shown on the right. The supply (usually the most positive voltage) is normally shown at the top of the diagram and the common, 0 V, or ground connection, is normally shown at the bottom. This rule is not always obeyed, particularly for complex diagrams where many signals and supply voltages may be present.

Note also that, in order to simplify a circuit diagram (and avoid having too many lines connected to the same point) multiple connections to common, 0 V, or ground may be shown using the appropriate symbol (see Fig. 1.24). The same applies to supply

---

**Example 1.26**

A coil of 800 turns is wound on a closed mild steel core having a length 600 mm and cross-sectional area 500 mm\(^2\). Determine the current required to establish a flux of 0.8 mWb in the core.

**Solution**

Now \( B = \frac{\Phi}{A} = \frac{0.8 \times 10^{-3}}{500 \times 10^{-6}} = 1.6 \text{ T} \)

From Fig. 1.17, a flux density of 1.6 T will occur in mild steel when \( H = 3,500 \text{ A/m} \). The current can
connections that may be repeated (appropriately labelled) at various points in the diagram.

A very simple circuit diagram (a simple resistance tester) is shown in Fig. 1.19. This circuit may be a little daunting if you haven’t met a circuit like it before, but you can still glean a great deal of information from the diagram even if you don’t know what the individual components do.

The circuit uses two batteries, B1 (a 9 V multi-cell battery) and B2 (a 1.5 V single-cell battery). The two batteries are selected by means of a double-pole, double-throw (DPDT) switch. This allows the circuit to operate from either the 9 V battery (B1) as shown in Fig. 1.19(a) or from the 1.5 V battery (B2) as shown in Fig. 1.19(b), depending on the setting of S1.

A variable resistor, VR1, is used to adjust the current supplied by whichever of the two batteries is currently selected. This current flows first through VR1, then through the milliammeter, and finally through the unknown resistor, \( R_X \). Notice how the meter terminals are labelled showing their polarity (the current flows into the positive terminal and out of the negative terminal).

The circuit shown in Fig. 1.19(c) uses a different type of switch but provides exactly the same function. In this circuit a single-pole, double-throw (SPDT) switch is used and the negative connections to the two batteries are ‘commoned’ (i.e. connected directly together).

Finally, Fig. 1.19(d) shows how the circuit can be re-drawn using a common ‘chassis’ connection to provide the negative connection between \( R_X \) and the two batteries. Electrically this circuit is identical to the one shown in Fig. 1.19(c).

**Practical investigation**

**Objective**

To investigate the relationship between the resistance in a circuit and the current flowing in it.

**Components and test equipment**

Breadboard, digital or analogue meter with d.c. current ranges, 9 V d.c. power source (either a 9V battery or an a.c. mains adapter with a 9 V 400 mA output), test leads, resistors of
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100 Ω, 220 Ω, 330 Ω, 470 Ω, 680 Ω and 1kΩ, connecting wire.

Procedure

Connect the circuit as shown in Fig. 1.21 and Fig. 1.22. Before switching on the d.c. supply or connecting the battery, check that the meter is set to the 200 mA d.c. current range. Switch on (or connect the battery), switch the multimeter on and read the current. Note down the current in the table below and repeat for resistance values of 220 Ω, 330 Ω, 470 Ω, 680 Ω and 1kΩ, switching off or disconnecting the battery between each measurement. Plot corresponding values of current (on the vertical axis) against resistance (on the horizontal axis) (see Fig. 1.23).

Measurements

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>Current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td></td>
</tr>
<tr>
<td>330</td>
<td></td>
</tr>
<tr>
<td>470</td>
<td></td>
</tr>
<tr>
<td>680</td>
<td></td>
</tr>
<tr>
<td>1k</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

Comment on the shape of the graph. Is this what you would expect and does it confirm that the current flowing in the circuit is inversely proportional to the resistance in the circuit?
Finally, use Ohm’s Law to calculate the value of each resistor and compare this with the marked value (but before doing this, you might find it useful to make an accurate measurement of the d.c. supply or battery voltage).

**Important formulae introduced in this chapter**

Voltage, current and resistance (Ohm’s Law):

\[ V = I R \]

Resistance and resistivity:

\[ R = \rho \frac{l}{A} \]

Charge, current and time:

\[ Q = I t \]

Power, current and voltage:

\[ P = I V \]

Power, voltage and resistance:

\[ P = \frac{V^2}{R} \]

Power, current and resistance:

\[ P = \frac{1}{2} R \]

Reluctance and permeability:

\[ S = l / \mu A \]

Flux and flux density:

\[ B = \Phi / A \]

Current and magnetic field intensity:

\[ H = N I / l \]

Flux, current and reluctance:

\[ N I = S\Phi \]
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Problems

1.1 Which of the following are not fundamental units; amperes, metres, coulombs, joules, hertz, kilogram?

1.2 A commonly used unit of consumer energy is the kilowatt hour (kWh). Express this in joules (J).

1.3 Express an angle of 30° in radians.

1.4 Express an angle of 0.2 radians in degrees.

1.5 A resistor has a value of 39,570 Ω. Express this in kilohms (kΩ).

1.6 An inductor has a value of 680 mH. Express this in henries (H).

1.7 A capacitor has a value of 0.00245 μF. Express this in nanofarads (nF).

1.8 A current of 190 μA is applied to a circuit. Express this in milliamperes (mA).

1.9 A signal of 0.475 mV appears at the input of an amplifier. Express this in volts using exponent notation.

1.10 A cable has an insulation resistance of 16.5 MΩ. Express this resistance in ohms using exponent notation.

1.11 Perform the following arithmetic using exponents:
(a) \((1.2 \times 10^3) \times (4 \times 10^3)\)
(b) \((3.6 \times 10^6) \div (2 \times 10^{-3})\)
(c) \((4.8 \times 10^8) + (1.2 \times 10^8)\)
(d) \((9.9 \times 10^{-6}) + (19.8 \times 10^{-3})\)
(e) \((4 \times 10^7) \times (7.5 \times 10^5) \times (2.5 \times 10^{-9})\)

1.12 Which one of the following metals is the best conductor of electricity: aluminium, copper, silver, or mild steel? Why?

1.13 A resistor of 270 Ω is connected across a 9 V d.c. supply. What current will flow?

1.14 A current of 56 μA flows in a 120 kΩ resistor. What voltage drop will appear across the resistor?

1.15 A voltage drop of 13.2 V appears across a resistor when a current of 4 mA flows in it. What is the value of the resistor?

1.16 A power supply is rated at 15 V, 1 A. What value of load resistor would be required to test the power supply at its full rated output?

1.17 A wirewound resistor is made from a 4 m length of aluminium wire \((ρ = 2.18 \times 10^{-8} \text{ Ωm})\). Determine the resistance of the wire if it has a cross-sectional area of 0.2 mm².

1.18 A current of 25 mA flows in a 47 Ω resistor. What power is dissipated in the resistor?

1.19 A 9 V battery supplies a circuit with a current of 75 mA. What power is consumed by the circuit?

1.20 A resistor of 150 Ω is rated at 0.5 W. What is the maximum current that can be applied to the resistor without exceeding its rating?

1.21 Determine the electric field strength that appears in the space between two parallel plates separated by an air gap of 4 mm if a potential of 2.5 kV exists between them.

1.22 Determine the current that must be applied to a straight wire conductor in order to produce a flux density of 200 μT at a distance of 12 mm in free space.

1.23 A flux density of 1.2 mT is developed in free space over an area of 50 cm². Determine the total flux present.

1.24 A ferrite rod has a length of 250 mm and a diameter of 10 mm. Determine the reluctance if the rod has a relative permeability of 2,500.

1.25 A coil of 400 turns is wound on a closed mild steel core having a length of 400 mm and cross-sectional area of 480 mm².

Figure 1.25 See Questions 1.26 and 1.27
1.26 Determine the current required to establish a flux of 0.6 mWb in the core.

1.27 Identify the type of switch shown in Fig. 1.25.

1.27 Fig. 1.25 shows a simple voltmeter. If the milliammeter reads 1 mA full-scale and has negligible resistance, determine the values for $R_1$ to $R_4$ that will provide voltage ranges of 1 V, 3 V, 10 V and 30 V full-scale.

Answers to these problems appear on page 416.
Passive components

Chapter summary

This chapter introduces several of the most common types of electronic component, including resistors, capacitors and inductors. These are often referred to as **passive components** as they cannot, by themselves, generate voltage or current. An understanding of the characteristics and application of passive components is an essential prerequisite to understanding the operation of the circuits used in amplifiers, oscillators, filters and power supplies.
Resistors

The notion of resistance as opposition to current was discussed in the previous chapter. Conventional forms of resistor obey a straight line law when voltage is plotted against current (see Fig. 2.1) and this allows us to use resistors as a means of converting current into a corresponding voltage drop, and vice versa (note that doubling the applied current will produce double the voltage drop, and so on). Therefore resistors provide us with a means of controlling the currents and voltages present in electronic circuits. They can also act as loads to simulate the presence of a circuit during testing (e.g. a suitably rated resistor can be used to replace a loudspeaker when an audio amplifier is being tested).

The specifications for a resistor usually include the value of resistance expressed in ohms (\(\Omega\)), kilohms (k\(\Omega\)) or megohms (M\(\Omega\)), the accuracy or tolerance (quoted as the maximum permissible percentage deviation from the marked value) and the power rating (which must be equal to, or greater than, the maximum expected power dissipation).

Other practical considerations when selecting resistors for use in a particular application include temperature coefficient, noise performance, stability and ambient temperature range. Table 2.1 summarizes the properties of five of the most common types of resistor. Fig. 2.2 shows a typical selection of fixed resistors with values from 15 \(\Omega\) to 4.7 k\(\Omega\).

Preferred values

The value marked on the body of a resistor is not its exact resistance. Some minor variation in resistance value is inevitable due to production tolerance. For example, a resistor marked 100 \(\Omega\) and produced within a tolerance of \(\pm 10\%\) will have a value which falls within the range 90 \(\Omega\) to 110 \(\Omega\). A similar component with a tolerance of \(\pm 1\%\) would have a value that falls within the range 99 \(\Omega\) to 101 \(\Omega\). Thus, where accuracy is important it is essential to use close tolerance components.

Resistors are available in several series of fixed decade values, the number of values provided with each series being governed by the tolerance involved. In order to cover the full range of resistance values using resistors having a \(\pm 20\%\) tolerance it will be necessary to provide six basic values (known as the E6 series). More values will be required in the series which offers a \(\pm 10\%\) tolerance. The E12 series provides 12 basic values. The E24 series for resistors of \(\pm 5\%\) tolerance provides no fewer than 24 basic values and, as with the E6 and E12 series, decade multiples (i.e. \(\times 1\, \times 10\, \times 100\, \times 1k\, \times 10k\, \times 100k\) and \(\times 1M\)) of the basic series. Fig. 2.3 shows the relationship between the E6, E12 and E24 series.

Power ratings

Resistor power ratings are related to operating temperatures and resistors should be derated at high temperatures. Where reliability is important
2 Passive components

Table 2.1 Characteristics of common types of resistor

<table>
<thead>
<tr>
<th>Property</th>
<th>Carbon film</th>
<th>Metal film</th>
<th>Metal oxide</th>
<th>Ceramic wirewound</th>
<th>Vitreous wirewound</th>
<th>Metal clad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance range (Ω)</td>
<td>10 to 10 M</td>
<td>1 to 1 M</td>
<td>10 to 10 M</td>
<td>0.47 to 22 k</td>
<td>0.1 to 22 k</td>
<td>0.05 to 10k</td>
</tr>
<tr>
<td>Typical tolerance (%)</td>
<td>±5</td>
<td>±1</td>
<td>±2</td>
<td>±5</td>
<td>±5</td>
<td>±5</td>
</tr>
<tr>
<td>Power rating (W)</td>
<td>0.25 to 2</td>
<td>0.125 to 0.5</td>
<td>0.25 to 0.5</td>
<td>4 to 17</td>
<td>2 to 4</td>
<td>10 to 300</td>
</tr>
<tr>
<td>Temperature coefficient (ppm/°C)</td>
<td>−250</td>
<td>+50 to +100</td>
<td>+250</td>
<td>+250</td>
<td>+75</td>
<td>+50</td>
</tr>
<tr>
<td>Stability</td>
<td>Fair</td>
<td>Excellent</td>
<td>Excellent</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Noise performance</td>
<td>Fair</td>
<td>Excellent</td>
<td>Excellent</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>Ambient temperature range (°C)</td>
<td>−45 to +125</td>
<td>−45 to +125</td>
<td>−45 to +125</td>
<td>−45 to +125</td>
<td>−45 to +125</td>
<td>−55 to +200</td>
</tr>
<tr>
<td>Typical applications</td>
<td>General purpose</td>
<td>Amplifiers, test equipment, etc.; requiring low-noise high-tolerance components</td>
<td>Power supplies, loads, medium- and high-power applications</td>
<td>Very high power applications</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.3 The E6, E12 and E24 series resisters should be operated at well below their nominal maximum power dissipation.

Example 2.1

A resistor has a marked value of 220 Ω. Determine the tolerance of the resistor if it has a measured value of 207 Ω.

Solution

The difference between the marked and measured values of resistance (the error) is (220 Ω − 207 Ω) = 13 Ω. The tolerance is given by:

\[
\text{Tolerance} = \frac{\text{error}}{\text{marked value}} \times 100\%
\]

The tolerance is thus \((13 / 220) \times 100 = 5.9\%\).

Example 2.2

A 9 V power supply is to be tested with a 39 Ω load resistor. If the resistor has a tolerance of 10% find:

(a) the nominal current taken from the supply;
(b) the maximum and minimum values of supply current at either end of the tolerance range for the resistor.
Solution

(a) If a resistor of exactly 39 Ω is used the current will be:
\[ I = \frac{V}{R} = \frac{9 \text{ V}}{39 \Omega} = 231 \text{ mA} \]

(b) The lowest value of resistance would be
\[ (39 \Omega - 3.9 \Omega) = 35.1 \Omega \] In which case the current would be:
\[ I = \frac{V}{R} = \frac{9 \text{ V}}{35.1 \Omega} = 256.4 \text{ mA} \]

At the other extreme, the highest value would be
\[ (39 \Omega + 3.9 \Omega) = 42.9 \Omega \]

In this case the current would be:
\[ I = \frac{V}{R} = \frac{9 \text{ V}}{42.9 \Omega} = 209.8 \text{ mA} \]

The maximum and minimum values of supply current will thus be 256.4 mA and 209.8 mA, respectively.

Example 2.3

A current of 100 mA (±20%) is to be drawn from a 28 V d.c. supply. What value and type of resistor should be used in this application?

Solution

The value of resistance required must first be calculated using Ohm’s Law:
\[ R = \frac{V}{I} = \frac{28 \text{ V}}{100 \text{ mA}} = 280 \Omega \]

The nearest preferred value from the E12 series is 270 Ω (which will actually produce a current of 103.7 mA (i.e. within ±4% of the desired value). If a resistor of ±10% tolerance is used, current will be within the range 94 mA to 115 mA (well within the ±20% accuracy specified).

The power dissipated in the resistor (calculated using \( P = I \times V \)) will be 2.9 W and thus a component rated at 3 W (or more) will be required. This would normally be a vitreous enamel-coated wirewound resistor (see Table 2.1).

Resistor markings

Carbon and metal oxide resistors are normally marked with colour codes which indicate their value and tolerance. Two methods of colour coding are in common use; one involves four coloured bands (see Fig. 2.4) while the other uses five colour bands (see Fig. 2.5).

Example 2.4

A resistor is marked with the following coloured stripes: brown, black, red, silver. What is its value and tolerance?

Solution

See Fig. 2.6.
2 Passive components

**Example 2.5**
A resistor is marked with the following coloured stripes: red, violet, orange, gold. What is its value and tolerance?

**Solution**
See Fig. 2.7.

**Example 2.6**
A resistor is marked with the following coloured stripes: green, blue, black, gold. What is its value and tolerance?

**Solution**
See Fig. 2.8.

**Example 2.7**
A resistor is marked with the following coloured stripes: red, green, black, black, brown. What is its value and tolerance?

**Solution**
See Fig. 2.9.

**Example 2.8**
A 2.2 kΩ of ±2% tolerance is required. What four-band colour code does this correspond to?

**Solution**
Red (2), red (2), red (2 zeros), red (2% tolerance). Thus all four bands should be red.
Example 2.9
A resistor is marked coded with the legend 4R7K. What is its value and tolerance?

Solution
4.7 Ω ± 10%

Example 2.10
A resistor is marked coded with the legend 330RG. What is its value and tolerance?

Solution
330 Ω ± 2%

Example 2.11
A resistor is marked coded with the legend R22M. What is its value and tolerance?

Solution
0.22 Ω ± 20%

Series and parallel combinations of resistors
In order to obtain a particular value of resistance, fixed resistors may be arranged in either series or parallel as shown in Figs 2.10 and 2.11. The effective resistance of each of the series circuits shown in Fig. 2.10 is simply equal to the sum of the individual resistances. So, for the circuit shown in Fig. 2.10(a):

$$R = R_1 + R_2$$

while for Fig. 2.10(b)

$$R = R_1 + R_2 + R_3$$

Turning to the parallel resistors shown in Fig. 2.11, the reciprocal of the effective resistance of each circuit is equal to the sum of the reciprocals of the individual resistances. Hence, for Fig. 2.11(a):

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

while for Fig. 2.12(b)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In the former case, the formula can be more conveniently re-arranged as follows:

BS 1852 coding
Some types of resistor have markings based on a system of coding defined in BS 1852. This system involves marking the position of the decimal point with a letter to indicate the multiplier concerned, as shown in Table 2.2. A further letter is then appended to indicate the tolerance, as shown in Table 2.3.

Table 2.2 BS 1852 resistor multiplier markings

<table>
<thead>
<tr>
<th>Letter</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1</td>
</tr>
<tr>
<td>K</td>
<td>1,000</td>
</tr>
<tr>
<td>M</td>
<td>1,000,000</td>
</tr>
</tbody>
</table>

Table 2.3 BS 1852 resistor tolerance markings

<table>
<thead>
<tr>
<th>Letter</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>±1%</td>
</tr>
<tr>
<td>G</td>
<td>±2%</td>
</tr>
<tr>
<td>J</td>
<td>±5%</td>
</tr>
<tr>
<td>K</td>
<td>±10%</td>
</tr>
<tr>
<td>M</td>
<td>±20%</td>
</tr>
</tbody>
</table>
2 Passive components

\[ R = \frac{R_1 \times R_2}{R_1 + R_2} \]

You can remember this as the product of the two resistance values divided by the sum of the two resistance values.

**Example 2.12**
Resistors of 22 \(\Omega\), 47 \(\Omega\) and 33 \(\Omega\) are connected (a) in series and (b) in parallel. Determine the effective resistance in each case.

**(a)**
\[ R = 22 + 47 + 33 = 102 \Omega \]

**(b)**
\[ \frac{1}{R} = \frac{1}{22} + \frac{1}{47} + \frac{1}{33} \]

thus

\[ \frac{1}{R} = 0.045 + 0.021 + 0.03 \]

from which

\[ \frac{1}{R} = 0.096 = 10.42 \Omega \]

**Example 2.13**
Determine the effective resistance of the circuit shown in Fig. 2.12.

**Solution**

(a) In the series circuit \( R = R_1 + R_2 + R_3 \), thus

\[ R = 22 \Omega + 47 \Omega + 33 \Omega = 102 \Omega \]

(b) In the parallel circuit:

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

thus

\[ \frac{1}{R} = \frac{1}{22} + \frac{1}{47} + \frac{1}{33} \]

or

\[ \frac{1}{R} = 0.045 + 0.021 + 0.03 \]

from which

\[ \frac{1}{R} = 0.096 = 10.42 \Omega \]

**Example 2.14**
A resistance of 50 \(\Omega\) rated at 2 W is required. What parallel combination of preferred value

You can remember this as the product of the two resistance values divided by the sum of the two resistance values.

**Example 2.12**
Resistors of 22 \(\Omega\), 47 \(\Omega\) and 33 \(\Omega\) are connected (a) in series and (b) in parallel. Determine the effective resistance in each case.

**(a)**
\[ R = 22 + 47 + 33 = 102 \Omega \]

**(b)**
\[ \frac{1}{R} = \frac{1}{22} + \frac{1}{47} + \frac{1}{33} \]

thus

\[ \frac{1}{R} = 0.045 + 0.021 + 0.03 \]

from which

\[ \frac{1}{R} = 0.096 = 10.42 \Omega \]

**Example 2.13**
Determine the effective resistance of the circuit shown in Fig. 2.12.

**Solution**

The circuit can be progressively simplified as shown in Fig. 2.13. The stages in this simplification are:

(a) \( R_1 \) and \( R_2 \) are in series and they can be replaced by a single resistance \( R_a \) of \((12 \Omega + 27 \Omega) = 39 \Omega \).

(b) \( R_a \) appears in parallel with \( R_3 \). These two resistors can be replaced by a single resistance \( R_b \) of \((39 \Omega \times 47 \Omega)/(39 \Omega + 47 \Omega) = 21.3 \Omega \).

(c) \( R_b \) appears in series with \( R_1 \). These two resistors can be replaced by a single resistance \( R \) of \((21.3 \Omega + 4.7 \Omega) = 26 \Omega \).

**Example 2.14**
A resistance of 50 \(\Omega\) rated at 2 W is required. What parallel combination of preferred value
resistors will satisfy this requirement? What power rating should each resistor have?

Solution
Two 100 \( \Omega \) resistors may be wired in parallel to provide a resistance of 50 \( \Omega \) as shown below:

\[
R = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{100 \times 100}{100 + 100} = \frac{10,000}{200} = 50 \Omega
\]

Note from this that when two resistors of the same value are connected in parallel the resulting resistance will be half that of a single resistor.

Having shown that two 100 \( \Omega \) resistors connected in parallel will provide us with a resistance of 50 \( \Omega \) we now need to consider the power rating. Since the resistors are identical, the applied power will be shared equally between them. Hence each resistor should have a power rating of 1 W.

Resistance and temperature

Fig. 2.14 shows how the resistance of a metal conductor (e.g. copper) varies with temperature. Since the resistance of the material increases with temperature, this characteristic is said to exhibit a positive temperature coefficient (PTC). Not all materials have a PTC characteristic.

The resistance of a carbon conductor falls with temperature and it is therefore said to exhibit a negative temperature coefficient (NTC).

The resistance of a conductor at a temperature, \( t \), is given by the equation:

\[
R_t = R_0 (1 + \alpha t + \beta t^2 + \gamma t^3 + \ldots)
\]

where \( \alpha \), \( \beta \), \( \gamma \), etc. are constants and \( R_0 \) is the resistance at 0 °C.

The coefficients, \( \beta \), \( \gamma \), etc. are quite small and since we are normally only dealing with a relatively restricted temperature range (e.g. 0 °C to 100 °C) we can usually approximate the characteristic shown in Fig. 2.14 to the straight line law shown in Fig. 2.15. In this case the equation simplifies to:

\[
R_t = R_0 (1 + \alpha t)
\]

where \( \alpha \) is known as the temperature coefficient of resistance. Table 2.4 shows some typical values for \( \alpha \) (note that \( \alpha \) is expressed in \( \Omega/\Omega/°C \) or just /°C).

Table 2.4 Temperature coefficient of resistance

<table>
<thead>
<tr>
<th>Material</th>
<th>Temperature coefficient of resistance, ( \alpha ) (/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platinum</td>
<td>+0.0034</td>
</tr>
<tr>
<td>Silver</td>
<td>+0.0038</td>
</tr>
<tr>
<td>Copper</td>
<td>+0.0043</td>
</tr>
<tr>
<td>Iron</td>
<td>+0.0065</td>
</tr>
<tr>
<td>Carbon</td>
<td>-0.0005</td>
</tr>
</tbody>
</table>

Example 2.15
A resistor has a temperature coefficient of 0.001/°C. If the resistor has a resistance of 1.5 kΩ at 0 °C, determine its resistance at 80 °C.

Solution
Now

\[
R_t = R_0 (1 + \alpha t)
\]

thus

\[
R_t = 1.5 \text{ kΩ} \times (1 + (0.001 \times 80))
\]

Hence

\[
R_t = 1.5 \times 1.08 = 1.62 \text{ kΩ}
\]
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\[ R_{90} = 673.3 \times (1 + (0.0005 \times 90)) \]

Hence
\[ R_{90} = 673.3 \times 1.045 = 704 \ \Omega \]

**Example 2.17**

A resistor has a resistance of 40 \( \Omega \) at 0°C and 44 \( \Omega \) at 100°C. Determine the resistor’s temperature coefficient.

**Solution**

First we need to make \( \alpha \) the subject of the formula:
\[ R_t = R_0(1 + \alpha t) \]

Now
\[ \alpha = \frac{1}{t} \left( \frac{R_t}{R_0} - 1 \right) = \frac{1}{100} \left( \frac{44}{40} - 1 \right) \]

from which
\[ \alpha = \frac{1}{100} (1.1 - 1) = \frac{1}{100} \times 0.1 = 0.001/°C \]

**Thermistors**

With conventional resistors we would normally require resistance to remain the same over a wide range of temperatures (i.e. \( \alpha \) should be zero). On the other hand, there are applications in which we could use the effect of varying resistance to detect a temperature change. Components that allow us to do this are known as thermistors. The resistance of a thermistor changes markedly with temperature and these components are widely used in temperature sensing and temperature compensating applications. Two basic types of thermistor are available, NTC and PTC (see Fig. 2.16).

Typical NTC thermistors have resistances that vary from a few hundred (or thousand) ohms at 25°C to a few tens (or hundreds) of ohms at 100°C. PTC thermistors, on the other hand, usually have a resistance–temperature characteristic which remains substantially flat (typically at around 100 \( \Omega \)) over the range 0°C to around 75°C. Above this, and at a critical temperature (usually in the range 80°C to 120°C)
Passive components

Light-dependent resistors

Light-dependent resistors (LDRs) use a semiconductor material (i.e. a material that is neither a conductor nor an insulator) whose electrical characteristics vary according to the amount of incident light. The two semiconductor materials used for the manufacture of LDRs are cadmium sulphide (CdS) and cadmium selenide (CdSe). These materials are most sensitive to light in the visible spectrum, peaking at about 0.6 µm for CdS and 0.75 µm for CdSe. A typical CdS LDR exhibits a resistance of around 1 MΩ in complete darkness and less than 1 kΩ when placed under a bright light source (see Fig. 2.17).

Voltage-dependent resistors

The resistance of a voltage-dependent resistor (VDR) falls very rapidly when the voltage across it exceeds a nominal value in either direction (see Fig. 2.18). In normal operation, the current flowing in a VDR is negligible; however, when the resistance falls, the current will become appreciable and a significant amount of energy will be absorbed. VDRs are used as a means of ‘clamping’ the voltage in a circuit to a predetermined level. When connected across the supply rails to a circuit (either a.c or d.c.) they are able to offer a measure of protection against voltage surges.
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Variable resistors

Variable resistors are available in several forms including those which use carbon tracks and those which use a wirewound resistance element. In either case, a moving slider makes contact with the resistance element. Most variable resistors have three (rather than two) terminals and as such are more correctly known as potentiometers. Carbon potentiometers are available with linear or semi-logarithmic law tracks (see Fig. 2.19) and in rotary or slider formats. Ganged controls, in which several potentiometers are linked together by a common control shaft, are also available. Fig. 2.20 shows a selection of variable resistors.

You will also encounter various forms of preset resistors that are used to make occasional adjustments (e.g. for calibration). Various forms of preset resistor are commonly used including open carbon track skeleton presets and fully encapsulated carbon and multi-turn cermet types, as shown in Fig. 2.21.

Capacitors

A capacitor is a device for storing electric charge. In effect, it is a reservoir into which charge can be deposited and then later extracted. Typical applications include reservoir and smoothing capacitors for use in power supplies, coupling a.c. signals between the stages of amplifiers, and decoupling supply rails (i.e. effectively grounding the supply rails as far as a.c. signals are concerned).

A capacitor can consist of nothing more than two parallel metal plates as shown in Fig. 1.10 on page 11. To understand what happens when a capacitor is being charged and discharged take a look at Fig. 2.22. If the switch is left open (position A), no charge will appear on the plates and in this condition there will be no electric field
in the space between the plates nor will there be any charge stored in the capacitor.

When the switch is moved to position B, electrons will be attracted from the positive plate to the positive terminal of the battery. At the same time, a similar number of electrons will move from the negative terminal of the battery to the negative plate. This sudden movement of electrons will manifest itself in a momentary surge of current (conventional current will flow from the positive terminal of the battery towards the positive terminal of the capacitor).

Eventually, enough electrons will have moved to make the e.m.f. between the plates the same as that of the battery. In this state, the capacitor is said to be **fully charged** and an electric field will be present in the space between the two plates.

If, at some later time, the switch is moved back to position A, the positive plate will be left with a deficiency of electrons while the negative plate will be left with a surplus of electrons. Furthermore, since there is no path for current to flow between the two plates the capacitor will remain charged and a potential difference will be maintained between the plates.

Now assume that the switch is moved to position C. The excess electrons on the negative plate will flow through the resistor to the positive plate until a neutral state once again exists (i.e. until there is no excess charge on either plate). In this state the capacitor is said to be **fully discharged** and the electric field between the plates will rapidly collapse. The movement of electrons during the discharging of the capacitor will again result in a momentary surge of current (current will flow from the positive terminal of the capacitor and into the resistor).

Fig. 2.23 shows the direction of current flow in the circuit of Fig. 2.22 during charging (switch in position B) and discharging (switch in position C). It should be noted that current flows momentarily in both circuits even though you may think that the circuit is broken by the gap between the capacitor plates!

![Figure 2.22 Capacitor charging and discharging](image-url)
Example 2.18
A voltage is changing at a uniform rate from 10 V to 50 V in a period of 0.1 s. If this voltage is applied to a capacitor of 22 μF, determine the current that will flow.

Solution
Now the current flowing will be given by:

\[ i = C \times \text{(rate of change of voltage)} \]

Thus

\[ i = 22 \times 10^{-6} \times \left( \frac{50 - 10}{0.1} \right) \]

From which

\[ i = 22 \times 10^{-6} \times \left( \frac{40}{0.1} \right) = 22 \times 10^{-6} \times 400 \]

so

\[ i = 8.8 \times 10^{-3} = 8.8 \text{ mA} \]

Charge, capacitance and voltage

The charge or quantity of electricity that can be stored in the electric field between the capacitor plates is proportional to the applied voltage and the capacitance of the capacitor. Thus:

\[ Q = CV \]

where \( Q \) is the charge (in coulombs), \( C \) is the capacitance (in farads) and \( V \) is the potential difference (in volts).

Example 2.19
A 10 uF capacitor is charged to a potential of 250 V. Determine the charge stored.

Solution
The charge stored will be given by:

\[ Q = CV = 10 \times 10^{-6} \times 250 = 2.5 \text{ mC} \]

Energy storage

The energy stored in a capacitor is proportional to the product of the capacitance and the square of the potential difference. Thus:

\[ W = \frac{1}{2} C V^2 \]
where \( W \) is the energy (in joules), \( C \) is the capacitance (in farads) and \( V \) is the potential difference (in volts).

**Example 2.20**
A capacitor of 47 \( \mu \)F is required to store 4 J of energy. Determine the potential difference that must be applied to the capacitor.

**Solution**
The foregoing formula can be re-arranged to make \( V \) the subject as follows:

\[
V = \sqrt{\frac{E}{0.5C}} = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2 \times 4}{47 \times 10^{-6}}}
\]

from which

\[
V = \sqrt{\frac{8}{47 \times 10^{-6}}} = \sqrt{0.170 \times 10^6} = 0.412 \times 10^3 = 412 \text{ V}
\]

**Capacitance and physical dimensions**
The capacitance of a capacitor depends upon the physical dimensions of the capacitor (i.e. the size of the plates and the separation between them) and the dielectric material between the plates. The capacitance of a conventional parallel plate capacitor is given by:

\[
C = \frac{\varepsilon_0 \varepsilon_r A}{d}
\]

where \( C \) is the capacitance (in farads), \( \varepsilon_0 \) is the permittivity of free space, \( \varepsilon_r \) is the relative permittivity of the dielectric medium between the plates and \( d \) is the separation between the plates (in metres).

**Example 2.21**
A capacitor of 1 nF is required. If a dielectric material of thickness 0.1 mm and relative permittivity 5.4 is available, determine the required plate area.

**Solution**
Re-arranging the formula

\[
C = \frac{\varepsilon_0 \varepsilon_r A}{d}
\]

to make \( A \) the subject gives:

\[
A = \frac{Cd}{\varepsilon_0 \varepsilon_r} = \frac{1 \times 10^{-3} \times 0.1 \times 10^{-3}}{8.854 \times 10^{-12} \times 5.4}
\]

from which

\[
A = \frac{0.1 \times 10^{-12}}{47.8116 \times 10^{-12}}
\]

thus

\[
A = 0.00209 \text{ m}^2 \text{ or } 20.9 \text{ cm}^2
\]

In order to increase the capacitance of a capacitor, many practical components employ multiple plates (see Fig. 2.24). The capacitance is then given by:

\[
C = \frac{\varepsilon_0 \varepsilon_r (n-1)A}{d}
\]

where \( C \) is the capacitance (in farads), \( \varepsilon_0 \) is the permittivity of free space, \( \varepsilon_r \) is the relative permittivity of the dielectric medium between the plates, \( d \) is the separation between the plates (in metres) and \( n \) is the total number of plates.

**Example 2.22**
A capacitor consists of six plates each of area 20 cm\(^2\) separated by a dielectric of relative permittivity 4.5 and thickness 0.2 mm. Determine the value of capacitance.

**Solution**
Using

\[
C = \frac{\varepsilon_0 \varepsilon_r (n-1)A}{d}
\]

gives:

\[
C = \frac{8.854 \times 10^{-12} \times 4.5 \times (6-1) \times 20 \times 10^{-4}}{0.2 \times 10^{-3}}
\]

from which

\[
C = \frac{3,984.3 \times 10^{-16}}{0.2 \times 10^{-3}} = 19.921 \times 10^{-10}
\]
2 Passive components

thus

\[ C = 1.992 \text{ nF} \]

**Capacitor specifications**

The specifications for a capacitor usually include the value of capacitance (expressed in microfarads, nanofarads or picofarads), the voltage rating (i.e. the maximum voltage which can be continuously applied to the capacitor under a given set of conditions) and the accuracy or tolerance (quoted as the maximum permissible percentage deviation from the marked value).

Other practical considerations when selecting capacitors for use in a particular application include temperature coefficient, leakage current, stability and ambient temperature range.

Table 2.5 summarizes the properties of five of the most common types of capacitor. Note that electrolytic capacitors require the application of a polarizing voltage in order to cause the chemical action on which they depend for their operation. The polarizing voltages used for electrolytic capacitors can range from as little as 1 V to several hundred volts depending upon the working voltage rating for the component in question.

Fig. 2.25 shows some typical non-electrolytic capacitors (including polyester, polystyrene, ceramic and mica types) while Fig. 2.26 shows a selection of electrolytic (polarized) capacitors.

An air-spaced variable capacitor is shown later in Fig. 2.34 on page 39.

---

**Table 2.5** Characteristics of common types of capacitor

<table>
<thead>
<tr>
<th>Property</th>
<th>Ceramic</th>
<th>Electrolytic</th>
<th>Polyester</th>
<th>Mica</th>
<th>Polystyrene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance range (F)</td>
<td>2.2 p to 100 n</td>
<td>100 n to 10 m</td>
<td>10 n to 2.2 μ</td>
<td>0.47 to 22 k</td>
<td>10 p to 22 n</td>
</tr>
<tr>
<td>Typical tolerance (%)</td>
<td>±10 and ±20</td>
<td>−10 to +50</td>
<td>±10</td>
<td>±1</td>
<td>±1</td>
</tr>
<tr>
<td>Typical voltage rating (V)</td>
<td>50 V to 200 V</td>
<td>6.3 V to 400 V</td>
<td>100 V to 400 V</td>
<td>350 V</td>
<td>100 V</td>
</tr>
<tr>
<td>Temperature coefficient (ppm/°C)</td>
<td>+100 to −4700</td>
<td>+1000 typical</td>
<td>+100 to +200</td>
<td>+50</td>
<td>+250</td>
</tr>
<tr>
<td>Stability</td>
<td>Fair</td>
<td>Poor</td>
<td>Good</td>
<td>Excellent</td>
<td>Good</td>
</tr>
<tr>
<td>Ambient temperature range (°C)</td>
<td>−85 to +85</td>
<td>−40 to +80</td>
<td>−40 to +100</td>
<td>−40 to +125</td>
<td>−40 to +100</td>
</tr>
<tr>
<td>Typical applications</td>
<td>High-frequency and low-cost</td>
<td>Smoothing and decoupling</td>
<td>General purpose</td>
<td>Tuned circuits and oscillators</td>
<td>General purpose</td>
</tr>
</tbody>
</table>

---

**Figure 2.25** A typical selection of non-electrolytic capacitors (including polyester, polystyrene, ceramic and mica types) with values ranging from 10 pF to 470 nF and working voltages from 50 V to 250 V

**Figure 2.26** A typical selection of electrolytic (polarized) capacitors with values ranging from 1 μF to 470 μF and working voltages from 10 V to 63 V
Capacitor markings

The vast majority of capacitors employ written markings which indicate their values, working voltages, and tolerance. The most usual method of marking resin-dipped polyester (and other) types of capacitor involves quoting the value (μF, nF or pF), the tolerance (often either 10% or 20%), and the working voltage (often using _ and ~ to indicate d.c. and a.c., respectively).

Several manufacturers use two separate lines for their capacitor markings and these have the following meanings (see Fig. 2.27):

First line: capacitance (pF or μF) and tolerance (K = 10%, M = 20%)

Second line: rated d.c. voltage and code for the dielectric material

A three-digit code is commonly used to mark monolithic ceramic capacitors. The first two digits of this code correspond to the first two digits of the value while the third digit is a multiplier which gives the number of zeros to be added to give the value in picofarads. Other capacitors may use a colour code similar to that used for marking resistor values (see Fig. 2.28).

Example 2.23
A monolithic ceramic capacitor is marked with the legend ‘103K’. What is its value?

Solution
The value (pF) will be given by the first two digits (10) followed by the number of zeros indicated by the third digit (3). The value of the capacitor is thus 10,000 pF or 10 nF. The final letter (K) indicates that the capacitor has a tolerance of 10%.

Example 2.24
A tubular capacitor is marked with the following coloured stripes: brown, green, brown, red, brown. What is its value, tolerance and working voltage?

Solution
See Fig. 2.29.
2 Passive components

Series and parallel combination of capacitors

In order to obtain a particular value of capacitance, fixed capacitors may be arranged in either series or parallel (Figs 2.30 and 2.31). The reciprocal of the effective capacitance of each of the series circuits shown in Fig. 2.30 is equal to the sum of the reciprocals of the individual capacitances. Hence, for Fig. 2.30(a):

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}
\]

while for Fig. 2.30(b):

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}
\]

In the former case, the formula can be more conveniently re-arranged as follows:

\[
C = \frac{C_1 \times C_2}{C_1 + C_2}
\]

You can remember this as the product of the two capacitor values divided by the sum of the two values – just as you did for two resistors in parallel.

For a parallel arrangement of capacitors, the effective capacitance of the circuit is simply equal to the sum of the individual capacitances. Hence, for Fig. 2.31(a):

\[
C = C_1 + C_2
\]

while for Fig. 2.31(b)

\[
C = C_1 + C_2 + C_3
\]

Example 2.25

Determine the effective capacitance of the circuit shown in Fig. 2.32.

Solution

The circuit of Fig. 2.32 can be progressively simplified as shown in Fig. 2.33. The stages in this simplification are:

(a) \(C_1\) and \(C_2\) are in parallel and they can be replaced by a single capacitor \((C_a)\) of \((2 \text{ nF} + 4 \text{ nF}) = 6 \text{ nF}\).

(b) \(C_a\) appears in series with \(C_3\). These two resistors can be replaced by a single capacitor \((C_b)\) of \((6 \text{ nF} \times 2 \text{ nF})/(6 \text{ nF} + 2 \text{ nF}) = 1.5 \text{ nF}\).

(c) \(C_b\) appears in parallel with \(C_4\). These two capacitors can be replaced by a single capacitance \((C)\) of \((1.5 \text{ nF} + 4 \text{ nF}) = 5.5 \text{ nF}\).
2 Passive components

by wiring equal, high-value (e.g. 100 kΩ) resistors across each capacitor.

Variable capacitors

By moving one set of plates relative to the other, a capacitor can be made variable. The dielectric material used in a variable capacitor can be either air (see Fig. 2.34) or plastic (the latter tends to be more compact). Typical values for variable capacitors tend to range from about 25 pF to 500 pF. These components are commonly used for tuning radio receivers.

Inductors

Inductors provide us with a means of storing electrical energy in the form of a magnetic field. Typical applications include chokes, filters and (in conjunction with one or more capacitors) frequency selective circuits. The electrical characteristics of an inductor are determined by a number of factors including the material of the core (if any), the number of turns and the physical dimensions of the coil. Fig. 2.35 shows the construction of a typical toroidal inductor wound on a ferrite (high permeability) core.

In practice every coil comprises both inductance (L) and a small resistance (R). The circuit of Fig. 2.36 shows these as two discrete components. In reality the inductance and the resistance (we often refer to this as a loss resistance because it’s something that we don’t actually want) are

Example 2.26

A capacitance of 50 μF (rated at 100 V) is required. What series combination of preferred value capacitors will satisfy this requirement? What voltage rating should each capacitor have?

Solution

Two 100 μF capacitors wired in series will provide a capacitance of 50 μF, as follows:

\[
C = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{100 \times 100}{100 + 100} = \frac{10,000}{200} = 50 \, \mu F
\]

Figure 2.33 See Example 2.25

Figure 2.34 An air-spaced variable capacitor. This component (used for tuning an AM radio) has two separate variable capacitors (each of 500 pF maximum) operated from a common control shaft
2 Passive components

Fig. 2.37(d), the magnetic field will suddenly collapse and the energy will be returned to the circuit in the form of an induced back e.m.f., which will appear across the coil as the field collapses. For large values of magnetic flux and inductance this back e.m.f. can be extremely large!

**Inductance**

Inductance is the property of a coil which gives rise to the opposition to a change in the value of current flowing in it. Any change in the current applied to a coil/inductor will result in an induced voltage appearing across it. The unit of inductance is the henry (H) and a coil is said to have an inductance of 1 H if a voltage of 1 V is induced across it when a current changing at the rate of 1 A/s is flowing in it.

The voltage induced across the terminals of an inductor will thus be proportional to the product of the inductance ($L$) and the rate of change of applied current. Hence:

$$e = -L \times \text{(rate of change of current)}$$

Note that the minus sign indicates the polarity of the voltage, i.e. opposition to the change.

The rate of change of current is often represented by the expression $\frac{di}{dt}$ where $di$ represents a very small change in current and $dt$ represents the corresponding small change in time. Using mathematical notation to write this we arrive at:

$$e = -L \frac{di}{dt}$$

You might like to compare this with the similar relationship that we obtained for the current flowing in a capacitor shown on page 34.

**Example 2.27**

A current increases at a uniform rate from 2 A to 6 A in a period of 250 ms. If this current is applied to an inductor of 600 mH, determine the voltage induced.

**Solution**

Now the induced voltage will be given by:

$$e = -L \times \text{(rate of change of current)}$$
2 Passive components

Thus

\[ e = -L \left( \frac{\text{change in current}}{\text{change in time}} \right) = -60 \times 10^{-3} \times \left( \frac{6 - 2}{250 \times 10^{-3}} \right) \]

From which

\[ e = -600 \times 10^{-3} \times \left( \frac{4}{0.25} \right) = -0.6 \times 10^{-3} \times 16 \]

so

\[ e = -9.6 \text{ V} \]

Energy storage

The energy stored in an inductor is proportional to the product of the inductance and the square of the current flowing in it. Thus:

\[ W = \frac{1}{2} L I^2 \]

where \( W \) is the energy (in joules), \( L \) is the capacitance (in henries) and \( I \) is the current flowing in the inductor (in amperes).

Example 2.28

An inductor of 20 mH is required to store 2.5 J of energy. Determine the current that must be applied.

Solution

The foregoing formula can be re-arranged to make \( I \) the subject as follows:

\[ I = \frac{E}{\sqrt{0.5L}} = \sqrt{\frac{2E}{L}} = \sqrt{\frac{2 \times 2.5}{20 \times 10^{-3}}} \]

From which

\[ I = \sqrt{\frac{5}{20 \times 10^{-3}}} = \sqrt{0.25 \times 10^{-3}} = \sqrt{250} = 15.81 \text{ A} \]

Inductance and physical dimensions

The inductance of an inductor depends upon the physical dimensions of the inductor (e.g. the length and diameter of the winding), the number of turns and the permeability of the material of the core. The inductance of an inductor is given by:

\[ L = \frac{\mu_0 \mu_n n^2 A}{l} \]
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where \( L \) is the inductance (in henries), \( \mu_0 \) is the permeability of free space, \( \mu_r \) is the relative permeability of the magnetic core, \( l \) is the mean length of the core (in metres) and \( A \) is the cross-sectional area of the core (in square metres).

**Example 2.29**

An inductor of 100 mH is required. If a closed magnetic core of length 20 cm, cross-sectional area 15 cm\(^2\) and relative permeability 500 is available, determine the number of turns required.

**Solution**

First we must re-arrange the formula

\[
L = \frac{\mu_0 \mu_r n^2 A}{l}
\]

in order to make \( n \) the subject:

\[
n = \sqrt{\frac{L \times l}{\mu_0 \mu_r n^2 A}} = \sqrt{\frac{100 \times 10^{-3} \times 20 \times 10^{-2}}{12.57 \times 10^{-7} \times 500 \times 15 \times 10^{-3}}}
\]

From which

\[
n = \frac{2 \times 10^{-2}}{94.275 \times 10^{-11}} = \sqrt{21215} = 146
\]

Hence the inductor requires 146 turns of wire.

**Inductor specifications**

Inductor specifications normally include the value of inductance (expressed in henries, millihenries or microhenries), the current rating (i.e. the maximum current which can be continuously applied to the inductor under a given set of conditions), and the accuracy or tolerance (quoted as the maximum permissible percentage deviation from the marked value). Other considerations may include the temperature coefficient of the inductance (usually expressed in parts per million, p.p.m., per unit temperature change), the stability of the inductor, the d.c. resistance of the coil windings (ideally zero), the Q-factor (quality factor) of the coil and the recommended working frequency range. Table 2.6 summarizes the properties of four common types of inductor. Some typical small inductors are shown in Fig. 2.38. These have values of inductance ranging from 15 \( \mu \)H to 1 mH.

**Inductor markings**

As with capacitors, the vast majority of inductors use written markings to indicate values, working current and tolerance. Some small inductors are marked with coloured stripes to indicate their value and tolerance (in which case the standard

<table>
<thead>
<tr>
<th>Property</th>
<th>Inductor type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core material</td>
<td>Air</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>50 n to 100 ( \mu )</td>
</tr>
<tr>
<td>Typical d.c. resistance (( \Omega ))</td>
<td>0.05 to 5</td>
</tr>
<tr>
<td>Typical tolerance (%)</td>
<td>( \pm 5 )</td>
</tr>
<tr>
<td>Typical Q-factor</td>
<td>60</td>
</tr>
<tr>
<td>Typical frequency range (Hz)</td>
<td>1 M to 500 M</td>
</tr>
<tr>
<td>Typical applications</td>
<td>Tuned circuits and filters</td>
</tr>
<tr>
<td></td>
<td>Filters and HF transformers</td>
</tr>
<tr>
<td></td>
<td>LF and MF filters and transformers</td>
</tr>
<tr>
<td></td>
<td>Smoothing chokes and filters</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>Ferrite rod</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>10 ( \mu ) to 1 m</td>
</tr>
<tr>
<td>Typical d.c. resistance (( \Omega ))</td>
<td>0.1 to 10</td>
</tr>
<tr>
<td>Typical tolerance (%)</td>
<td>( \pm 10 )</td>
</tr>
<tr>
<td>Typical Q-factor</td>
<td>80</td>
</tr>
<tr>
<td>Typical frequency range (Hz)</td>
<td>100 k to 100 M</td>
</tr>
<tr>
<td>Typical applications</td>
<td>Filters and HF transformers</td>
</tr>
<tr>
<td></td>
<td>LF and MF filters and transformers</td>
</tr>
<tr>
<td></td>
<td>Smoothing chokes and filters</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>Ferrite pot</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>1 m to 100 m</td>
</tr>
<tr>
<td>Typical d.c. resistance (( \Omega ))</td>
<td>5 to 100</td>
</tr>
<tr>
<td>Typical tolerance (%)</td>
<td>( \pm 10 )</td>
</tr>
<tr>
<td>Typical Q-factor</td>
<td>40</td>
</tr>
<tr>
<td>Typical frequency range (Hz)</td>
<td>1 k to 10 M</td>
</tr>
<tr>
<td>Typical applications</td>
<td>LF and MF filters and transformers</td>
</tr>
<tr>
<td></td>
<td>Smoothing chokes and filters</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>Iron cored</td>
</tr>
<tr>
<td>Inductance range (H)</td>
<td>20 m to 20</td>
</tr>
<tr>
<td>Typical d.c. resistance (( \Omega ))</td>
<td>10 to 200</td>
</tr>
<tr>
<td>Typical tolerance (%)</td>
<td>( \pm 20 )</td>
</tr>
<tr>
<td>Typical Q-factor</td>
<td>20</td>
</tr>
<tr>
<td>Typical frequency range (Hz)</td>
<td>50 to 10 k</td>
</tr>
<tr>
<td>Typical applications</td>
<td>Smoothing chokes and filters</td>
</tr>
</tbody>
</table>
colour values are used and inductance is normally expressed in microhenries).

**Series and parallel combinations of inductors**

In order to obtain a particular value of inductance, fixed inductors may be arranged in either series or parallel, as shown in Figs 2.39 and 2.40. The effective inductance of each of the series circuits shown in Fig. 2.39 is simply equal to the sum of the individual inductances. So, for the circuit shown in Fig. 2.39(a):

\[ L = L_1 + L_2 \]

while for Fig. 2.39(b)

\[ L = L_1 + L_2 + L_3 \]

Turning to the parallel inductors shown in Fig. 2.40, the reciprocal of the effective inductance of each circuit is equal to the sum of the reciprocals of the individual inductances. Hence, for Fig. 2.40(a):

\[ \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \]

while for Fig. 2.40(b)

\[ \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \]

In the former case, the formula can be more conveniently re-arranged as follows:

\[ L = \frac{L_1 \times L_2}{L_1 + L_2} \]

You can remember this as the product of the two inductance values divided by the sum of the two inductance values.

**Example 2.30**

An inductance of 5 mH (rated at 2 A) is required. What parallel combination of preferred value inductors will satisfy this requirement?

**Solution**

Two 10 mH inductors may be wired in parallel to provide an inductance of 5 mH as shown below:

\[ L = \frac{L_1 \times L_2}{L_1 + L_2} = \frac{10 \times 10}{10 + 10} = \frac{100}{20} = 5 \text{ mH} \]

Since the inductors are identical, the applied current will be shared equally between them. Hence each inductor should have a current rating of 1 A.
2 Passive components

Example 2.31
Determine the effective inductance of the circuit shown in Fig. 2.41.

Solution
The circuit can be progressively simplified as shown in Fig. 2.42. The stages in this simplification are as follows:

(a) \( L_1 \) and \( L_2 \) are in series and they can be replaced by a single inductance \( L_{eq} \) of \( (60 + 60) = 120 \text{ mH} \).

(b) \( L_3 \) appears in parallel with \( L_{eq} \). These two inductors can be replaced by a single inductor \( L_{tp} \) of \( (120 \times 120)/(120 + 120) = 60 \text{ mH} \).

(c) \( L_4 \) appears in series with \( L_{eq} \). These two inductors can be replaced by a single inductance \( L \) of \( (60 + 50) = 110 \text{ mH} \).

Variable inductors
A ferrite-cored inductor can be made variable by moving its core in or out of the former onto which the coil is wound. Many small inductors have threaded ferrite cores to make this possible (see Fig. 2.43). Such inductors are often used in radio and high-frequency applications where precise tuning is required.

Surface-mounted components (SMCs)
Surface-mounting technology (SMT) is now widely used in the manufacture of printed circuit boards (PCBs) for electronic equipment. SMT allows circuits to be assembled in a much smaller space than would be possible using components with conventional wire leads and pins that are mounted using through-hole techniques. It is also possible to mix the two technologies, i.e. some through-hole mounting of components and some SMCs present on the same circuit board. The following combinations are possible:

- SMCs on both sides of a printed circuit board
- SMC on one side of the board and conventional through-hole components (THCs) on the other
- A mixture of SMCs and THCs on both sides of the printed circuit board.

SMCs are supplied in packages that are designed for mounting directly on the surface of a PCB. To provide electrical contact with the PCB, some SMCs have contact pads on their surface. Other
Passive components

Devices have contacts which extend beyond the outline of the package itself but which terminate on the surface of the PCB rather than making contact through a hole (as is the case with a conventional THC). In general, passive components (such as resistors, capacitors and inductors) are configured leadless for surface mounting, while active devices (such as transistors and integrated circuits) are available in both surface mountable types as well as lead and leadless terminations suitable for making direct contact to the pads on the surface of a PCB.

Most SMCs have a flat rectangular shape rather than the cylindrical shape that we associate with conventional wire leaded components. During manufacture of a PCB, the various SMCs are attached using re-flow soldering paste (and in some cases adhesives) which consists of particles of solder and flux together with binder, solvents and additives. They need to have good ‘tack’ in order to hold the components in place and remove oxides without leaving obstinate residues.

The component attachment (i.e. soldering!) process is completed using one of several techniques including convection ovens in which the PCB is passed, using a conveyor belt, through a convection oven which has separate zones for preheating, flowing and cooling, and infra-red re-flow in which infra-red lamps are used to provide the source of heat.

SMCs are generally too small to be marked with colour codes. Instead, values may be marked using three digits. For example, the first two digits marked on a resistor normally specify the first two digits of the value while the third digit gives the number of zeros that should be added.

Example 2.32

In Fig. 2.45, R88 is marked ‘102’. What is its value?

Solution

R88 will have a value of 1,000 Ω (i.e. 10 followed by two zeros).
2 Passive components

Practical investigation

Objective
To investigate the resistance of series and parallel combinations of resistors.

Components and test equipment
Breadboard, digital or analogue meter with d.c. current ranges, 9 V d.c. power source (either a 9 V battery or an a.c. mains adapter with a 9 V 400 mA output), test leads, resistors of 100 Ω, 220 Ω, 330 Ω, 470 Ω, 680 Ω and 1 kΩ, connecting wire.

Procedure
Connect the resistor network and power supply (or battery) as shown in Fig. 2.46. Select the 20 V d.c. voltage range on the multimeter then measure and record the supply voltage (this should be approximately 9 V). Now break the positive connection to the circuit, change the range on the multimeter to the 20 mA d.c. current range and measure the current supplied to the circuit.

Next, measure and record the voltage dropped across each resistor (don’t forget to change ranges on the multimeter when making each measurement). Finally, break the circuit at one end of each resistor in turn, then measure and record the current flowing. Repeat the procedure for the other two resistor network circuits shown in Figs. 2.47 and 2.48.

Measurements and calculations
Record your results in a table for each network. Use the recorded values of current and voltage for each resistor to calculate the value of resistance and compare this with the marked value. Check that the measured value lies within the tolerance band for each resistor.

Calculate the resistance of each network (looking in at the supply terminals) and compare this with the resistance calculated by dividing the supply voltage by the supply current.

Conclusion
Comment on the results. Did your measured values agree with the marked values? Were these within the tolerance range for the resistors used in the investigation? If the readings were not in agreement can you suggest why?
Important formulae introduced in this chapter

Component tolerance:
(page 24)

\[
\text{Tolerance} = \frac{\text{error}}{\text{marked value}} \times 100\% 
\]

Resistors in series:
(page 27)
\[ R = R_1 + R_2 + R_3 \]
Resistors in parallel:
(page 27)
\[ R = \frac{R_1 R_2}{R_1 + R_2} \]

Two resistors in parallel:
(page 28)
\[ R = \frac{R_1 R_2}{R_1 + R_2} \]

Resistance and temperature:
(page 29)
\[ R_t = R_0 (1 + \alpha t) \]

Current flowing in a capacitor:
(page 34)
\[ i = \frac{C}{d} \frac{dV}{dt} \]

Charge in a capacitor:
(page 34)
\[ Q = CV \]

Energy stored in a capacitor:
(page 34)
\[ W = \frac{1}{2} CV^2 \]

Capacitance of a capacitor:
(page 35)
\[ C = \frac{\varepsilon_0 \varepsilon_r A}{d} \]

Capacitors in series:
(page 38)
\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]

Capacitors in parallel:
(page 38)
\[ C = \frac{C_1 C_2}{C_1 + C_2} \]

Induced e.m.f. in an inductor:
(page 40)
\[ e = -L \frac{di}{dt} \]

Energy stored in an inductor:
(page 41)
\[ W = \frac{1}{2} LI^2 \]

Inductance of an inductor:
(page 41)
\[ L = \frac{\mu_0 \mu_r r^2 A}{l} \]

Inductors in series:
(page 43)
\[ L = L_1 + L_2 + L_3 \]

Inductors in parallel:
(page 43)
\[ \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \]

Two inductors in parallel:
(page 43)
\[ L = \frac{L_1 L_2}{L_1 + L_2} \]
2 Passive components

Symbols introduced in this chapter

![Circuit symbols introduced in this chapter](image)

**Problems**

2.1 A power supply rated at 15 V, 0.25 A is to be tested at full rated output. What value of load resistance is required and what power rating should it have? What type of resistor is most suitable for this application and why?

2.2 Determine the value and tolerance of resistors marked with the following coloured bands:

(a) red, violet, yellow, gold  
(b) brown, black, black, silver  
(c) blue, grey, green, gold  
(d) orange, white, silver, gold  
(e) red, red, black, brown, red.

2.3 A batch of resistors are all marked yellow, violet, black, gold. If a resistor is selected from this batch, within what range would you expect its value to be?

2.4 Resistors of 27 Ω, 33 Ω, 56 Ω and 68 Ω are available. How can two or more of these be arranged to realize the following resistance values?

(a) 60 Ω  
(b) 14.9 Ω  
(c) 124 Ω  
(d) 11.7 Ω  
(e) 128 Ω.

2.5 Three 100 Ω resistors are connected as shown in Fig. 2.50. Determine the effective resistance of the circuit.

2.6 Determine the effective resistance of the circuit shown in Fig. 2.51.
2.12 Three 180 pF capacitors are connected (a) in series and (b) in parallel. Determine the effective capacitance of each combination.

2.13 Determine the effective capacitance of the circuit shown in Fig. 2.53.

2.14 A capacitor of 330 μF is charged to a potential of 63 V. Determine the quantity of energy stored.

2.15 A parallel plate capacitor has plates of 0.02 m². Determine the capacitance of the capacitor if the plates are separated by a dielectric of thickness 0.5 mm and relative permittivity 5.6.

2.16 A capacitor is required to store 0.5 J of energy when charged from a 120 V d.c. supply. Determine the value of capacitance required.

2.17 The current in a 2.5 H inductor increases uniformly from zero to 50 mA in 400 ms. Determine the e.m.f. induced.
2.18 An inductor has 200 turns of wire wound on a closed magnetic core of mean length 24 cm, cross-sectional area 10 cm$^2$ and relative permeability 650. Determine the inductance of the inductor.

2.19 A current of 4 A flows in a 60 mH inductor. Determine the energy stored.

2.20 Inductors of 22 mH and 68 mH are connected (a) in series and (b) in parallel. Determine the effective inductance in each case.

Answers to these problems appear on page 416.
D.C. circuits

Chapter summary
In many cases, Ohm’s Law alone is insufficient to determine the magnitude of the voltages and currents present in a circuit. This chapter introduces several techniques that simplify the task of solving complex circuits. It also introduces the concept of exponential growth and decay of voltage and current in circuits containing capacitance and resistance and inductance and resistance. It concludes by showing how humble C–R circuits can be used for shaping the waveforms found in electronic circuits. We start by introducing two of the most useful laws of electronics.
Kirchhoff’s Laws

Kirchhoff’s Laws relate to the algebraic sum of currents at a junction (or node) or voltages in a network (or mesh). The term ‘algebraic’ simply indicates that the polarity of each current or voltage drop must be taken into account by giving it an appropriate sign, either positive (+) or negative (−).

Kirchhoff’s Current Law states that the algebraic sum of the currents present at a junction (node) in a circuit is zero (see Fig. 3.1).

Example 3.1

In Fig. 3.2, use Kirchhoff’s Current Law to determine:
(a) the value of current flowing between A and B
(b) the value of $I_5$.

Solution

(a) $I_1$ and $I_2$ both flow towards Node A so, applying our polarity convention, they must both be positive. Now, assuming that current $I_5$ flows between A and B and that this current flows away from the junction (obvious because $I_1$ and $I_2$ both flow towards the junction) we arrive at the following Kirchhoff’s Current Law equation:

$$+I_1 + I_2 - I_5 = 0$$

From which:

$$I_5 = I_1 + I_2 = 1.5 + 2.7 = 4.2 \text{ A}$$

(b) Moving to Node B, let’s assume that $I_3$ flows outwards so we can say that:

$$+I_4 + I_5 - I_3 = 0$$

From which:

$$I_3 = I_4 + I_5 = 3.3 + 4.2 = 7.5 \text{ A}$$

Kirchhoff’s Voltage Law states that the algebraic sum of the potential drops in a closed network (or ‘mesh’) is zero (see Fig. 3.3).

Example 3.2

In Fig. 3.4, use Kirchhoff’s Voltage Law to determine:
(a) the value of $V_2$
(b) the value of $E_3$.

Solution

(a) $I_1$ and $I_2$ both flow towards Node A so, applying our polarity convention, they must both be positive. Now, assuming that current $I_5$ flows between A and B and that this current flows away from the junction (obvious because $I_1$ and $I_2$ both flow towards the junction) we arrive at the following Kirchhoff’s Current Law equation:

$$+I_1 + I_2 - I_5 = 0$$

From which:

$$I_5 = I_1 + I_2 = 1.5 + 2.7 = 4.2 \text{ A}$$

(b) Moving to Node B, let’s assume that $I_3$ flows outwards so we can say that:

$$+I_4 + I_5 - I_3 = 0$$

From which:

$$I_3 = I_4 + I_5 = 3.3 + 4.2 = 7.5 \text{ A}$$

Kirchhoff’s Voltage Law states that the algebraic sum of the potential drops in a closed network (or ‘mesh’) is zero (see Fig. 3.3).
3 D.C. circuits

Solution

(a) In Loop A, and using the conventions shown in Fig. 3.3, we can write down the Kirchhoff’s Voltage Law equations:

\[ E_1 - V_2 - E_2 = 0 \]

From which:

\[ V_2 = E_1 - E_2 = 6 - 3 = 3 \text{ V} \]

(b) Similarly, in Loop B we can say that:

\[ E_2 - V_1 + E_3 = 0 \]

From which:

\[ E_3 = V_1 - E_2 = 4.5 - 3 = 1.5 \text{ V} \]

Example 3.3

Determine the currents and voltages in the circuit of Fig. 3.5.

Solution

In order to solve the circuit shown in Fig. 3.5, it is first necessary to mark the currents and voltages on the circuit, as shown in Figs 3.6 and 3.7.

By applying Kirchhoff’s Current Law at Node A that we’ve identified in Fig. 3.5:

\[ +I_1 + I_2 - I_3 = 0 \]

Therefore:

\[ I_1 = I_3 - I_2 \] (i)

By applying Kirchhoff’s Voltage Law in Loop A we obtain:

\[ 12 - V_1 - V_3 = 0 \]

From which:

\[ V_1 = 12 - V_3 \] (ii)

By applying Kirchhoff’s Voltage Law in Loop B we obtain:

\[ 9 - V_2 - V_3 = 0 \]

From which:

\[ V_2 = 9 - V_3 \] (iii)

Next we can generate three further relationships by applying Ohm’s Law:

\[ V_1 = I_1 R_1 \quad \text{from which} \quad I_1 = \frac{V_1}{R_1} \]

\[ V_2 = I_2 R_2 \quad \text{from which} \quad I_2 = \frac{V_2}{R_2} \]

\[ V_3 = I_3 R_3 \quad \text{from which} \quad I_3 = \frac{V_3}{R_3} \]

Combining these three relationships with the Current Law equation (i) gives:

\[ \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3} \]

from which:

\[ \frac{V_1}{110} = \frac{V_2}{22} = \frac{V_3}{33} \] (iv)

Combining (ii) and (iii) with (iv) gives:

\[ \frac{(12 - V_3)}{110} = \frac{V_3}{22} - \frac{(9 - V_3)}{33} \]

Multiplying both sides of the expression gives:

\[ \frac{330(12 - V_3)}{110} = \frac{330V_3}{22} - \frac{330(9 - V_3)}{33} \]

\[ 3(12 - V_3) = 15V_3 - 10(9 - V_3) \]

From which:

\[ 36 - 3V_3 = 15V_3 - 90 + V_3 \]

\[ 36 + 90 = 15V_3 + 10V_3 + 3V_3 \]

and:

\[ 126 = 28V_3 \quad \text{so} \quad V_3 = \frac{126}{28} = 4.5 \text{ V} \]
Since the left- and right-hand sides of the equation are equal we can be reasonably confident that our results are correct.

### The potential divider

The potential divider circuit (see Fig. 3.8) is commonly used to reduce voltages in a circuit. The output voltage produced by the circuit is given by:

\[ V_{\text{out}} = V_n \frac{R_2}{R_1 + R_2} \]

It is, however, important to note that the output voltage \( V_{\text{out}} \) will fall when current is drawn from the arrangement.

Fig. 3.9 shows the effect of loading the potential divider circuit. In the loaded potential divider (Fig. 3.9) the output voltage is given by:

\[ V_{\text{out}} = V_n \frac{R_p}{R_1 + R_p} \]

where:

\[ R_p = \frac{R_2 \times R_L}{R_2 + R_L} \]

---

**Figure 3.6** See Example 3.3

**Figure 3.7** See Example 3.3

From (ii):

\[ V_1 = 12 - V_3 \quad \text{so} \quad V_1 = 12 - 4.5 = 7.5 \text{ V} \]

From (iii):

\[ V_2 = 9 - V_3 \quad \text{so} \quad V_2 = 9 - 4.5 = 4.5 \text{ V} \]

Using the Ohm’s Law equations that we met earlier gives:

\[ I_1 = \frac{V_1}{R_1} = \frac{7.5}{110} = 0.068 \text{ A} = 68 \text{ mA} \]

\[ I_2 = \frac{V_2}{R_2} = \frac{4.5}{33} = 0.136 \text{ A} = 136 \text{ mA} \]

\[ I_3 = \frac{V_3}{R_3} = \frac{4.5}{22} = 0.204 \text{ A} = 204 \text{ mA} \]

Finally, it’s worth checking these results with the Current Law equation (i):

\[ +I_1 + I_2 - I_3 = 0 \]

Inserting our values for \( I_1 \), \( I_2 \), and \( I_3 \) gives:

\[ +0.068 + 0.136 - 204 = 0 \]
flowing in a circuit. The output current produced by the circuit is given by:

\[ I_{\text{out}} = I_{\text{in}} \frac{R_1}{R_1 + R_2} \]

It is, however, important to note that the output current \( I_{\text{out}} \) will fall when the load connected to the output terminals has any appreciable resistance.

**Example 3.5**

A moving coil meter requires a current of 1 mA to provide full-scale deflection. If the meter coil has a resistance of 100 \( \Omega \) and is to be used as a milliammeter reading 5 mA full-scale, determine the value of parallel shunt resistor required.

**Solution**

This problem may sound a little complicated so it is worth taking a look at the equivalent circuit of the meter (Fig. 3.12) and comparing it with the current divider shown in Fig. 3.11.

We can apply the current divider formula, replacing \( I_{\text{out}} \) with \( I_m \) (the meter full-scale

**The current divider**

The current divider circuit (see Fig. 3.11) is used to divert a known proportion of the current

![Figure 3.10 See Example 3.4](image)

![Figure 3.11 Current divider circuit](image)

![Figure 3.12 See Example 3.5](image)
deflection current) and \( R_s \) with \( R_m \) (the meter resistance). \( R_s \) is the required value of shunt resistor, \( R_s \). Hence:

\[
I_{\text{out}} = I_{\text{in}} \frac{R_s}{R_s + R_m}
\]

Re-arranging the formula gives:

\[
I_{\text{in}} (R_s + R_m) = I_{\text{in}} R_s
\]

thus

\[
I_{\text{in}} R_s + I_{\text{in}} R_m = I_{\text{in}} R_s
\]

or

\[
I_{\text{in}} R_s - I_{\text{in}} R_m = I_{\text{in}} R_m
\]

from which

\[
R_s (I_{\text{in}} - I_m) = I_{\text{in}} R_m
\]

so

\[
R_s = \frac{I_{\text{in}} R_m}{I_{\text{in}} - I_m}
\]

Now \( I_{\text{in}} = 1 \text{ mA}, \) \( R_m = 100 \Omega \) and \( I_{\text{in}} = 5 \text{ mA}, \) thus:

\[
R_s = \frac{1 \times 100}{5 - 1} = \frac{100}{4} = 25 \Omega
\]

The Wheatstone bridge

The Wheatstone bridge forms the basis of a number of useful electronic circuits including several that are used in instrumentation and measurement.

The basic form of Wheatstone bridge is shown in Fig. 3.13. The voltage developed between A and B will be zero when the voltage between A and Y is the same as that between B and Y. In effect, \( R_1 \) and \( R_2 \) constitute a potential divider, as do \( R_3 \) and \( R_4 \).

The bridge will be balanced (and \( V_{\text{AB}} = 0 \)) when the ratio of \( R_1 : R_2 \) is the same as the ratio \( R_3 : R_4 \). Hence, at balance:

\[
\frac{R_1}{R_2} = \frac{R_3}{R_4}
\]

A practical form of Wheatstone bridge that can be used for measuring unknown resistances is shown in Fig. 3.14.

In this practical form of Wheatstone bridge, \( R_1 \) and \( R_2 \) are called the ratio arms while one arm (that occupied by \( R_3 \) in Fig. 3.13) is replaced by a calibrated variable resistor. The unknown resistor, \( R_x \), is connected in the fourth arm. At balance:

\[
\frac{R_1}{R_2} = \frac{R_x}{R_m} \quad \text{thus} \quad R_x = \frac{R_2}{R_1} R_m
\]

Example 3.6

A Wheatstone bridge is based on the circuit shown in Fig. 3.14. If \( R_1 \) and \( R_2 \) can each be switched so that they have values of either 100 Ω or 1 kΩ and \( R_v \) is variable between 10 Ω and 10 kΩ, determine the range of resistance values that can be measured.

Solution

The maximum value of resistance that can be measured will correspond to the largest ratio of \( R_2 : R_1 \) (i.e. when \( R_2 \) is 1 kΩ and \( R_1 \) is 100 Ω) and the highest value of \( R_v \) (i.e. 10 kΩ). In this case:

\[
R_x = \frac{1000 \times 10,000}{100,000} = 100,000 = 100 \text{ kΩ}
\]

The minimum value of resistance that can be measured will correspond to the smallest ratio of \( R_2 : R_1 \) (i.e. when \( R_1 \) is 100 Ω and \( R_2 \) is 1 kΩ) and the smallest value of \( R_v \) (i.e. 10 Ω). In this case:
3 D.C. circuits

The voltage across \( R_4 \) will be given by:

\[
V = 10 \times \frac{R_4}{R_3 + R_4} = 10 \times \frac{400}{500 + 400} = 4.444 \text{ V}
\]

Hence the voltage at B relative to Y, \( V_{BY} \), will be 4.444 V.

The voltage \( V_{AB} \) will be the difference between \( V_{AY} \) and \( V_{BY} \). This, the open-circuit output voltage, \( V_{AB} \), will be given by:

\[
V_{AB} = V_{AY} - V_{BY} = 5.454 - 4.444 = 1.01 \text{ V}
\]

Next we need to find the Thévenin equivalent resistance looking in at A and B. To do this, we can redraw the circuit, replacing the battery (connected between X and Y) with a short-circuit, as shown in Fig. 3.17.

The Thévenin equivalent resistance is given by the relationship:

\[
R_{\text{TH}} = \frac{R_1 \times R_2}{R_1 + R_2} + \frac{R_3 \times R_4}{R_3 + R_4} = \frac{500 \times 600}{500 + 600} + \frac{500 \times 400}{500 + 400}
\]

Hence the range of values that can be measured extends from 1 \( \Omega \) to 100 k\( \Omega \).

**Thévenin’s Theorem**

Thévenin’s Theorem allows us to replace a complicated network of resistances and voltage sources with a simple equivalent circuit comprising a single **voltage source** connected in series with a single resistance (see Fig. 3.15).

The single voltage source in the Thévenin equivalent circuit, \( V_{oc} \), is simply the voltage that appears between the terminals when nothing is connected to it. In other words, it is the open-circuit voltage that would appear between A and B.

The single resistance that appears in the Thévenin equivalent circuit, \( R \), is the resistance that would be seen looking into the network between A and B when all of the voltage sources (assumed perfect) are replaced by short-circuit connections. Note that if the voltage sources are not perfect (i.e. if they have some internal resistance) the equivalent circuit must be constructed on the basis that each voltage source is replaced by its own internal resistance.

Once we have values for \( V_{oc} \) and \( R \), we can determine how the network will behave when it is connected to a load (i.e. when a resistor is connected across the terminals A and B).

**Example 3.7**

Fig. 3.16 shows a Wheatstone bridge. Determine the current that will flow in a 100 \( \Omega \) load connected between terminals A and B.

**Solution**

First we need to find the Thévenin equivalent of the circuit. To find \( V_{oc} \) we can treat the bridge arrangement as two potential dividers.

The voltage across \( R_2 \) will be given by:

\[
V = 10 \times \frac{R_2}{R_1 + R_2} = 10 \times \frac{600}{500 + 600} = 5.454 \text{ V}
\]

Hence the voltage at A relative to Y, \( V_{AY} \), will be 5.454 V.

**Figure 3.15** Thévenin equivalent circuit

**Figure 3.16** See Example 3.7

**Figure 3.17** See Example 3.7
3 D.C. circuits

From which:

\[ R = \frac{300,000}{1,100} + \frac{200,000}{900} = 272.7 + 222.2 = 494.9 \, \Omega \]

The Thévenin equivalent circuit is shown in Fig. 3.18. To determine the current in a 100 \( \Omega \) load connected between A and B, we can simply add a 100 \( \Omega \) load to the Thévenin equivalent circuit, as shown in Fig. 3.19. By applying Ohm’s Law in Fig. 3.19 we get:

\[ I = \frac{V_{sc}}{R + 100} = \frac{1.01}{494.9 + 100} = \frac{1.01}{594.9} = 1.698 \, mA \]

Norton’s Theorem

Norton’s Theorem provides an alternative method of reducing a complex network to a simple equivalent circuit. Unlike Thévenin’s Theorem, Norton’s Theorem makes use of a current source rather than a voltage source. The Norton equivalent circuit allows us to replace a complicated network of resistances and voltage sources with a simple equivalent circuit comprising a single constant current source connected in parallel with a single resistance (see Fig. 3.20).

The constant current source in the Norton equivalent circuit, \( I_{sc} \), is simply the short-circuit current that would flow if A and B were to be linked directly together. The resistance that appears in the Norton equivalent circuit, \( R \), is the resistance that would be seen looking into the network between A and B when all of the voltage sources are replaced by short-circuit connections. Once again, it is worth noting that if the voltage sources have any appreciable internal resistance, the equivalent circuit must be constructed on the basis that each voltage source is replaced by its own internal resistance.

As with the Thévenin equivalent, we can determine how a network will behave by obtaining values for \( I_{sc} \) and \( R \).

Example 3.8

Three temperature sensors having the following characteristics shown in the table below are connected in parallel, as shown in Fig. 3.21.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output voltage (open circuit)</td>
<td>20 mV</td>
<td>30 mV</td>
<td>10 mV</td>
</tr>
<tr>
<td>Internal resistance</td>
<td>5 kΩ</td>
<td>3 kΩ</td>
<td>2 kΩ</td>
</tr>
</tbody>
</table>

Determine the voltage produced when the arrangement is connected to a moving-coil meter having a resistance of 1 kΩ.
The voltage appearing across the moving coil meter in Fig. 3.25 will be given by:

\[ V = I_{sc} \times \frac{R \times R_m}{R + R_m} = 19 \mu A \times \frac{1,000 \times 968}{1,000 + 968} \]

hence:

\[ V = 19 \mu A \times 492 \Omega = 9.35 \text{ mV} \]

**Figure 3.22** Determining the equivalent resistance in Fig. 3.21

**Figure 3.23** Norton equivalent of the circuit in Fig. 3.21

**Figure 3.24** Determining the output voltage when the Norton equivalent circuit is loaded with 1 kΩ

**Figure 3.25** The voltage drop across the meter is found to be 9.35 mV
3. D.C. circuits

**C–R circuits**

Networks of capacitors and resistors (known as C–R circuits) form the basis of many timing and pulse-shaping circuits and are thus often found in practical electronic circuits.

**Charging**

A simple C–R circuit is shown in Fig. 3.26. In this circuit C is charged through R from a constant voltage source, $V_s$. The voltage, $v_c$, across the (initially uncharged) capacitor voltage will rise exponentially as shown in Fig. 3.27. At the same time, the current in the circuit, $i$, will fall, as shown in Fig. 3.28.

The rate of growth of voltage with time (and decay of current with time) will be dependent upon the product of capacitance and resistance. This value is known as the **time constant** of the circuit. Hence:

$$t = CR$$

where $C$ is the value of capacitance (F), $R$ is the resistance (Ω) and $t$ is the time constant (s).

The voltage developed across the charging capacitor, $v_c$, varies with time, $t$, according to the relationship:

$$v_c = V_s \left(1 - e^{-\frac{t}{CR}}\right)$$

where $v_c$ is the capacitor voltage, $V_s$ is the d.c. supply voltage, $t$ is the time and $CR$ is the time constant of the circuit (equal to the product of capacitance, $C$, and resistance, $R$).

The capacitor voltage will rise to approximately 63% of the supply voltage, $V_s$, in a time interval equal to the time constant.

At the end of the next interval of time equal to the time constant (i.e. after an elapsed time equal to $2CR$) the voltage will have risen by 63% of the remainder, and so on. In theory, the capacitor will **never** become fully charged. However, after a period of time equal to $5CR$, the capacitor voltage will to all intents and purposes be equal to the supply voltage. At this point the capacitor voltage will have risen to 99.3% of its final value and we can consider it to be fully charged.
During charging, the current in the capacitor, \( i \), varies with time, \( t \), according to the relationship:
\[
i = \frac{V_s}{R} e^{-\frac{t}{CR}}
\]
where \( V_s \) is the d.c. supply voltage, \( t \) is the time, \( R \) is the series resistance and \( C \) is the value of capacitance.

The current will fall to approximately 37% of the initial current in a time equal to the time constant. At the end of the next interval of time equal to the time constant (i.e. after a total time of \( 2CR \) has elapsed) the current will have fallen by a further 37% of the remainder, and so on.

**Example 3.9**

An initially uncharged 1 \( \mu \)F capacitor is charged from a 9 V d.c. supply via a 3.3 M\( \Omega \) resistor. Determine the capacitor voltage 1 s after connecting the supply.

**Solution**

The formula for exponential growth of voltage in the capacitor is:
\[
\nu_c = V_s \left(1 - e^{-\frac{t}{CR}}\right)
\]

Here we need to find the capacitor voltage, \( \nu_c \), when \( V_s = 9 \text{ V}, t = 1 \text{ s}, C = 1 \ \mu \text{F} \) and \( R = 3.3 \ \text{M}\Omega \).

The time constant, \( CR \), will be given by:
\[
CR = 1 \times 10^{-6} \times 3.3 \times 10^6 = 3.3 \text{ s}
\]

Thus:
\[
\nu_c = 9 \left(1 - e^{-\frac{1}{3.3}}\right)
\]

and
\[
\nu_c = 9(1 - 0.738) = 9 \times 0.262 = 2.358 \text{ V}
\]

**Example 3.10**

A 100 \( \mu \)F capacitor is charged from a 350 V d.c. supply through a series resistance of 1 k\( \Omega \). Determine the initial charging current and the current that will flow 50 ms and 100 ms after connecting the supply. After what time is the capacitor considered to be fully charged?

**Solution**

At \( t = 0 \) the capacitor will be uncharged (\( \nu_c = 0 \)) and all of the supply voltage will appear across the series resistance. Thus, at \( t = 0 \):
\[
i = \frac{V_s}{R} = \frac{350}{1000} = 0.35 \text{ A}
\]

When \( t = 50 \text{ ms} \), the current will be given by:
\[
i = \frac{V_s}{R} e^{-\frac{t}{CR}}
\]

where \( V_s = 350 \text{ V}, t = 50 \text{ ms}, C = 100 \ \mu \text{F} \) and \( R = 1 \ \text{k}\Omega \). Hence:
\[
i = \frac{350}{1000} e^{-\frac{0.05}{3.3}} = 0.35 \times 0.607 = 0.21 \text{ A}
\]

When \( t = 100 \text{ ms} \) (using the same equation but with \( t = 0.1 \text{ s} \)) the current is given by:
\[
i = \frac{350}{1000} e^{-\frac{0.1}{3.3}} = 0.35 \times 0.368 = 0.129 \text{ A}
\]

The capacitor can be considered to be fully charged when \( t = 5CR = 5 \times 100 \times 10^{-6} \times 1 \times 10^3 = 0.5 \text{ s} \). Note that, at this point, the capacitor voltage will have reached 99% of its final value.

**Figure 3.29** C–R circuits are widely used in electronics. In this oscilloscope, for example, a rotary switch is used to select different C–R combinations in order to provide the various timebase ranges (adjustable from 500 ms/cm to 1 \( \mu \)s/cm). Each C–R time constant corresponds to a different timebase range.
Discharge

Having considered the situation when a capacitor is being charged, let’s consider what happens when an already charged capacitor is discharged.

When the fully charged capacitor from Fig. 3.24 is connected as shown in Fig. 3.30, the capacitor will discharge through the resistor, and the capacitor voltage, \( V_C \), will fall exponentially with time, as shown in Fig. 3.31.

The current in the circuit, \( i \), will also fall, as shown in Fig. 3.32. The rate of discharge (i.e. the rate of decay of voltage with time) will once again be governed by the time constant of the circuit, \( C \times R \).

The voltage developed across the discharging capacitor, \( V_C \), varies with time, \( t \), according to the relationship:

\[
V_C = V_s e^{-\frac{t}{CR}}
\]

where \( V_s \) is the supply voltage, \( t \) is the time, \( C \) is the capacitance and \( R \) is the resistance.

The capacitor voltage will fall to approximately 37% of the initial voltage in a time equal to the time constant. At the end of the next interval of time equal to the time constant (i.e. after an elapsed time equal to \( 2CR \)) the voltage will have fallen by 37% of the remainder, and so on.

In theory, the capacitor will never become fully discharged. However, after a period of time equal to \( 5CR \), the capacitor voltage will to all intents and purposes be zero.

At this point the capacitor voltage will have fallen below 1% of its initial value. We can consider it to be fully discharged.

As with charging, the current in the capacitor, \( i \), varies with time, \( t \), according to the relationship:

\[
i = \frac{V_s}{R} e^{-\frac{t}{CR}}
\]

where \( V_s \) is the supply voltage, \( t \) is the time, \( C \) is the capacitance and \( R \) is the resistance.

The current will fall to approximately 37% of the initial value of current, \( V_s / R \), in a time equal to the time constant.

At the end of the next interval of time equal to the time constant (i.e. after a total time of \( 2CR \) has
elapsed) the voltage will have fallen by a further 37% of the remainder, and so on.

**Example 3.11**

A 10 μF capacitor is charged to a potential of 20 V and then discharged through a 47 kΩ resistor. Determine the time taken for the capacitor voltage to fall below 10 V.

**Solution**

The formula for exponential decay of voltage in the capacitor is:

\[ V_c = V_s e^{-\frac{t}{CR}} \]

where \( V_s = 20 \text{ V} \) and \( CR = 10 \mu\text{F} \times 47 \text{ kΩ} = 0.47 \text{ s} \).

We need to find \( t \) when \( V_c = 10 \text{ V} \). Re-arranging the formula to make \( t \) the subject gives:

\[ t = -CR \ln \left( \frac{V_c}{V_s} \right) \]

thus

\[ t = -0.47 \times \ln \left( \frac{10}{20} \right) = -0.47 \times \ln(0.5) \]

or

\[ t = -0.47 \times -0.693 = 0.325 \text{ s} \]

In order to simplify the mathematics of exponential growth and decay, Table 3.1 provides an alternative tabular method that may be used to determine the voltage and current in a C–R circuit.

**Example 3.12**

A 150 μF capacitor is charged to a potential of 150 V. The capacitor is then removed from the charging source and connected to a 2 MΩ resistor. Determine the capacitor voltage 1 minute later.

**Solution**

We will solve this problem using Table 3.1 rather than the exponential formula.

First we need to find the time constant:

\[ C \times R = 150 \mu\text{F} \times 2 \text{ MΩ} = 300 \text{ s} \]

Next we need to find the ratio of \( t \) to \( CR \):

After 1 minute, \( t = 60 \text{ s} \) therefore the ratio of \( t \) to \( CR \) is 60/300 or 0.2. Table 3.1 shows that when \( t/CR = 0.2 \), the ratio of instantaneous value to final value (\( k \) in Table 3.1) is 0.8187.

Thus

\[ \frac{V_c}{V_s} = 0.8187 \]

or

\[ V_c = 0.8187 \times V_s = 0.8187 \times 150 \text{ V} = 122.8 \text{ V} \]

<table>
<thead>
<tr>
<th>( t/CR ) or ( t/(L/R) )</th>
<th>( k ) (growth)</th>
<th>( k ) (decay)</th>
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<td>0.8187 (1)</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>5.0</td>
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</tr>
</tbody>
</table>

Notes:

(1) See Example 3.12
(2) See Example 3.16

\( k \) is the ratio of the value at time, \( t \), to the final value (e.g. \( \frac{V_c}{V_s} \))
3 D.C. circuits

Waveshaping with C–R networks

One of the most common applications of C–R networks is in waveshaping circuits. The circuits shown in Figs 3.33 and 3.35 function as simple square-to-triangle and square-to-pulse converters by, respectively, integrating and differentiating their inputs.

The effectiveness of the simple integrator circuit shown in Fig. 3.33 depends on the ratio of time constant, $C \times R$, to periodic time, $t$. The larger this ratio is, the more effective the circuit will be as an integrator. The effectiveness of the circuit of Fig. 3.33 is illustrated by the input and output waveforms shown in Fig. 3.34.

Similarly, the effectiveness of the simple differentiator circuit shown in Fig. 3.35 also depends on the ratio of time constant $C \times R$, to periodic time, $t$. The smaller this ratio is, the more effective the circuit will be as a differentiator.

The effectiveness of the circuit of Fig. 3.35 is illustrated by the input and output waveforms shown in Fig. 3.36.

Example 3.13

A circuit is required to produce a train of alternating positive and negative pulses of short duration from a square wave of frequency 1 kHz. Devise a suitable C–R circuit and specify suitable values.

Solution

Here we require the services of a differentiating circuit along the lines of that shown in Fig. 3.35. In order that the circuit operates effectively as a differentiator, we need to make the time constant, $C \times R$, very much less than the periodic time of the input waveform (1 ms).

Assuming that we choose a medium value for $R$ of, say, 10 kΩ, the maximum value which
we could allow $C$ to have would be that which satisfies the equation:

$$C \times R = 0.1 \ t$$

where $R = 10 \ k\Omega$ and $t = 1 \ ms$. Thus

$$C = \frac{0.1 \ t}{R} = \frac{0.1 \times 1 \ ms}{10 \ k\Omega} = 0.1 \times 10^{-3} \times 10^{-4} = 1 \times 10^{-8} \ F$$

or

$$C = 10 \times 10^{-3} \ F = 10 \ nF$$

In practice, any value equal or less than 10 nF would be adequate. A very small value (say less than 1 nF) will, however, generate pulses of a very narrow width.

**Example 3.14**

A circuit is required to produce a triangular waveform from a square wave of frequency 1 kHz. Devise a suitable $C$-$R$ arrangement and specify suitable values.

**Solution**

This time we require an integrating circuit like that shown in Fig. 3.33. In order that the circuit operates effectively as an integrator, we need to make the time constant, $C \times R$, very much less than the periodic time of the input waveform (1 ms).

Assuming that we choose a medium value for $R$ of, say, 10 k$\Omega$, the minimum value which we could allow $C$ to have would be that which satisfies the equation:

$$C \times R = 10 \ t$$

where $R = 10 \ k\Omega$ and $t = 1 \ ms$. Thus

$$C = \frac{10 \ t}{R} = \frac{10 \times 1 \ ms}{10 \ k\Omega} = 10 \times 10^{-3} \times 10^{-4} = 1 \times 10^{-6} \ F$$

or

$$C = 1 \times 10^{-6} \ F = 1 \ \mu F$$

In practice, any value equal or greater than 1 $\mu F$ would be adequate. A very large value (say, more than 10 $\mu F$) will, however, generate a triangular wave which has a very small amplitude. To put this in simple terms, although the waveform might be what you want there’s not a lot of it!

**L–R circuits**

Networks of inductors and resistors (known as L–R circuits) can also be used for timing and pulse shaping. In comparison with capacitors, however, inductors are somewhat more difficult to manufacture and are consequently more expensive.

Inductors are also prone to losses and may also require screening to minimize the effects of stray magnetic coupling. Inductors are, therefore, generally unsuited to simple timing and waveshaping applications.

Fig. 3.37 shows a simple L–R network in which an inductor is connected to a constant voltage supply. When the supply is first connected, the current, $i$, will rise exponentially with time, as shown in Fig. 3.38. At the same time, the inductor voltage, $V_L$, will fall, as shown in Fig. 3.39. The rate of change of current with time will depend upon the ratio of inductance to resistance and is known as the time constant. Hence:

$$\text{Time constant, } t = \frac{L}{R}$$

where $L$ is the value of inductance (H), $R$ is the resistance ($\Omega$), and $t$ is the time constant (s).

The current flowing in the inductor, $i$, varies with time, $t$, according to the relationship:

$$i = \frac{V_s}{R} \left(1 - e^{-\frac{t}{L/R}}\right)$$

where $V_s$ is the d.c. supply voltage, $R$ is the resistance of the inductor and $L$ is the inductance.

The current, $i$, will initially be zero and will rise to approximately 63% of its maximum value (i.e. $V_s/R$) in a time interval equal to the time constant. At the end of the next interval of time equal to the time constant (i.e. after a total time of $2L/R$ has elapsed) the current will have risen by a further 63% of the remainder, and so on.

In theory, the current in the inductor will never become equal to $V_s/R$. However, after a period of time equal to $5L/R$, the current will to all intents and purposes be equal to $V_s/R$. At this point the current in the inductor will have risen to 99.3% of its final value.
The voltage developed across the inductor, $v_L$, varies with time, $t$, according to the relationship:

$$v_L = V_s e^{\frac{-t}{T}}$$

where $V_s$ is the d.c. supply voltage, $R$ is the resistance of the inductor and $L$ is the inductance.

The inductor voltage will fall to approximately 37% of the initial voltage in a time equal to the time constant.

At the end of the next interval of time equal to the time constant (i.e. after a total time of $2L/R$ has elapsed) the voltage will have fallen by a further 37% of the remainder, and so on.

**Example 3.15**

A coil having inductance 6 H and resistance 24 Ω is connected to a 12 V d.c. supply. Determine the current in the inductor 0.1 s after the supply is first connected.

**Solution**

The formula for exponential growth of current in the coil is:

$$i = \frac{V_s}{R} \left(1 - e^{\frac{-t}{T}}\right)$$

where $V_s = 12$ V, $L = 6$ H and $R = 24$ Ω.

We need to find $i$ when $t = 0.1$ s

$$i = \frac{12}{24} \left(1 - e^{\frac{-0.1\times24}{6}}\right) = 0.5(1 - e^{-0.4}) = 0.5(1 - 0.67)$$

thus

$$i = 0.5 \times 0.33 = 0.165 \text{ A}$$

In order to simplify the mathematics of exponential growth and decay, Table 3.1 provides an alternative tabular method that may be used to determine the voltage and current in an $L$–$R$ circuit.

**Example 3.16**

A coil has an inductance of 100 mH and a resistance of 10 Ω. If the inductor is connected to a 5 V d.c. supply, determine the inductor voltage 20 ms after the supply is first connected.
Solution
We will solve this problem using Table 3.1 rather than the exponential formula.
First we need to find the time constant:
\[ \frac{L}{R} = \frac{0.1 \text{ H}}{10 \text{ k} \Omega} = 0.01 \text{ s} \]
Next we find the ratio of \( t \) to \( \frac{L}{R} \).
When \( t = 20 \text{ ms} \) the ratio of \( t \) to \( \frac{L}{R} \) is 0.02/0.01 or 2. Table 3.1 shows that when \( \frac{t}{(L/R)} = 2 \), the ratio of instantaneous value to final value (\( k \)) is 0.8647. Thus
\[ \frac{V_L}{V_s} = 0.8647 \]
or
\[ V_L = 0.8647 \times V_s = 0.8647 \times 5 \text{ V} = 4.32 \text{ V} \]

Practical investigation
Objective
To investigate the charge and discharge of a capacitor.

Components and test equipment
Breadboard, 9 V d.c. power source (either a PP9 9 V battery or an a.c. mains adapter with a 9 V 400 mA output), digital multimeter with test leads, resistors of 100 k\( \Omega \), 220 k\( \Omega \) and 47 k\( \Omega \), capacitor of 1,000 \( \mu \text{F} \), insulated wire links (various lengths), assorted crocodile leads, short lengths of black, red and green insulated solid wire. A watch or clock with a seconds display will also be required for timing.

Procedure
Connect the charging circuit shown in Fig. 3.40 with \( R = 100 \text{ k} \Omega \) and \( C = 1,000 \mu \text{F} \). Place a temporary shorting link across the capacitor. Set the meter to the 20 V d.c. range and remove the shorting link. Measure and record the capacitor voltage at 25 s intervals over the range 0 to 250 s after removing the shorting link. Record your result in a table showing capacitor voltage against time. Repeat with \( R = 220 \text{ k} \Omega \) and \( R = 47 \text{ k} \Omega \).

Measure and record the capacitor voltage at 25 s intervals over the range 0 to 250 s from removing the link. Record your result in a table showing capacitor voltage against time. Repeat with \( R = 220 \text{ k} \Omega \) and \( R = 47 \text{ k} \Omega \).
3 D.C. circuits

Measurements and calculations

Plot graphs of voltage (on the vertical axis) against time (on the horizontal axis) using the graph layout shown in Fig. 3.42.

Calculate the time constant for each combination of resistance and capacitance that you used in the investigation.

Conclusion

Comment on the shape of the graphs. Is this what you would expect? For each combination of resistance and capacitance, estimate the time constant from the graph. Compare these values with the calculated values. If they are not the same suggest possible reasons for the difference.

Formulae introduced in this chapter

Kirchhoff’s Current Law:
(page 52)

Algebraic sum of currents = 0

Kirchhoff’s Voltage Law:
(page 52)

Algebraic sum of e.m.f.s = algebraic sum of voltage drops

Potential divider:
(page 54)

\[ V_{out} = V_{in} \frac{R_2}{R_1 + R_2} \]

Current divider:
(page 55)

\[ I_{out} = I_{in} \frac{R_1}{R_1 + R_2} \]

Wheatstone bridge:
(page 56)

\[ \frac{R_1}{R_2} = \frac{R_3}{R_4} \]

and

\[ R_x = \frac{R_2}{R_1} \times R_v \]

Time constant of a C–R circuit:
(page 60)

\[ t = CR \]

Capacitor voltage (charge):
(page 60)

\[ v_c = V_s \left(1 - e^{-\frac{1}{CR}}\right) \]

Capacitor current (charge):
(page 61)

\[ i = V_s e^{-\frac{1}{CR}} \]

Capacitor voltage (discharge):
(page 62)

\[ v_c = V_s e^{\frac{1}{CR}} \]

Capacitor current (discharge):
(page 62)

\[ i = V_s e^{\frac{1}{CR}} \]

Time constant of an L–R circuit:
(page 65)

\[ t = L / R \]

Inductor current (growth):
(page 65)

\[ i = \frac{V_s}{R} \left(1 - e^{-\frac{R}{L}}\right) \]

Inductor voltage (decay):
(page 66)

\[ v_L = V_s e^{\frac{R}{L}} \]
3. D.C. circuits

Symbols introduced in this chapter

![Symbols](image)

Figure 3.43 Circuit symbols introduced in this chapter

Problems

3.1 A power supply is rated at 500 mA maximum output current. If the supply delivers 150 mA to one circuit and 75 mA to another, how much current would be available for a third circuit?

3.2 A 15 V d.c. supply delivers a total current of 300 mA. If this current is shared equally between four circuits, determine the resistance of each circuit.

3.3 Determine the unknown current in each circuit shown in Fig. 3.44.

3.4 Determine the unknown voltage in each circuit shown in Fig. 3.45.

3.5 Determine all currents and voltages in Fig. 3.46.

3.6 Two resistors, one of 120 Ω and one of 680 Ω, are connected as a potential divider across a 12 V supply. Determine the voltage developed across each resistor.

3.7 Two resistors, one of 15 Ω and one of 5 Ω, are connected in parallel. If a current of 2 A is applied to the combination, determine the current flowing in each resistor.

3.8 A switched attenuator comprises five 1 kΩ resistors wired in series across a 5 V d.c. supply. If the output voltage is selected by means of a single-pole four-way switch, sketch a circuit and determine the voltage produced for each switch position.
3.16 The Norton equivalent of a network is shown in Fig. 3.50. Determine (a) the open-circuit output voltage and (b) the output voltage developed across a load of 5 kΩ.

Answers to these problems appear on page 416.
Alternating voltage and current

Chapter summary

This chapter introduces basic alternating current theory. We discuss the terminology used to describe alternating waveforms and the behaviour of resistors, capacitors and inductors when an alternating current is applied to them. The chapter concludes by introducing another useful component, the transformer.
4 Alternating voltage and current

Alternating versus direct current

Direct currents are currents which, even though their magnitude may vary, essentially flow only in one direction. In other words, direct currents are unidirectional. Alternating currents, on the other hand, are bidirectional and continuously reverse their direction of flow. The polarity of the e.m.f. which produces an alternating current must consequently also be changing from positive to negative, and vice versa.

Alternating currents produce alternating potential differences (voltages) in the circuits in which they flow. Furthermore, in some circuits alternating voltages may be superimposed on direct voltage levels (see Fig. 4.1). The resulting voltage may be unipolar (i.e. always positive or always negative) or bipolar (i.e. partly positive and partly negative).

Waveforms and signals

A graph showing the variation of voltage or current present in a circuit is known as a waveform. There are many common types of waveform encountered in electrical circuits, including sine (or sinusoidal), square, triangle, ramp or sawtooth (which may be either positive or negative going), and pulse.

Complex waveforms, like speech and music, usually comprise many components at different frequencies. Pulse waveforms are often categorized as either repetitive or non-repetitive (the former comprises a pattern of pulses that repeats regularly while the latter comprises pulses which constitute a unique event). Some common waveforms are shown in Fig. 4.2.

Signals can be conveyed using one or more of the properties of a waveform and sent using wires, cables, optical and radio links. Signals can also be processed in various ways using amplifiers, modulators, filters, etc. Signals are also classified as either analogue (continuously variable) or digital (based on discrete states).

Figure 4.1 (a) Bipolar sine wave; (b) unipolar sine wave (superimposed on a d.c. level)

Figure 4.2 Common waveforms

Frequency

The frequency of a repetitive waveform is the number of cycles of the waveform which occur in unit time. Frequency is expressed in hertz, Hz, and a frequency of 1 Hz is equivalent to one cycle per second. Hence, if a voltage has a frequency of 400 Hz, 400 cycles of it will occur in every second.
The equation for the voltage shown in Fig. 4.1(a) at a time, \( t \), is:
\[ \nu = V_{\text{max}} \sin (2\pi ft) \]
while that in Fig. 4.1(b) is:
\[ \nu = V_{\text{d.c.}} + V_{\text{max}} \sin (2\pi ft) \]
where \( \nu \) is the instantaneous voltage, \( V_{\text{max}} \) is the maximum (or peak) voltage of the sine wave, \( V_{\text{d.c.}} \) is the d.c. offset (where present) and \( f \) is the frequency of the sine wave.

**Example 4.1**
A sine wave voltage has a maximum value of 20 V and a frequency of 50 Hz. Determine the instantaneous voltage present (a) 2.5 ms and (b) 15 ms from the start of the cycle.

**Solution**
We can find the voltage at any instant of time using:
\[ \nu = V_{\text{max}} \sin (2\pi ft) \]
where \( V_{\text{max}} = 20 \text{ V} \) and \( f = 50 \text{ Hz} \).

In (a), \( t = 2.5 \text{ ms} \), hence:
\[ \nu = 20 \sin (2\pi \times 50 \times 0.0025) = 20 \sin (0.785) = 20 \times 0.707 = 14.14 \text{ V} \]
In (b), \( t = 15 \text{ ms} \), hence:
\[ \nu = 20 \sin (2\pi \times 50 \times 0.0015) = 20 \sin (4.71) = 20 \times -1 = -20 \text{ V} \]

**Periodic time**
The periodic time (or **period**) of a waveform is the time taken for one complete cycle of the wave (see Fig. 4.3). The relationship between periodic time and frequency is thus:
\[ t = \frac{1}{f} \text{ or } f = \frac{1}{t} \]
where \( t \) is the periodic time (in s) and \( f \) is the frequency (in Hz).

**Example 4.2**
A waveform has a frequency of 400 Hz. What is the periodic time of the waveform?

**Solution**
\[ t = \frac{1}{f} = \frac{1}{400} = 0.0025 \text{ s} \text{ (or } 2.5 \text{ ms)} \]

**Average, peak, peak–peak, and r.m.s. values**
The **average value** of an alternating current which swings symmetrically above and below zero will be zero when measured over a long period of time. Hence average values of currents and voltages are invariably taken over one complete half-cycle (either positive or negative) rather than over one complete full-cycle (which would result in an average value of zero).

The **amplitude** (or **peak value**) of a waveform is a measure of the extent of its voltage or current excursion from the resting value (usually zero). The **peak-to-peak value** for a wave which is symmetrical about its resting value is twice its peak value (see Fig. 4.4).

The **r.m.s.** (or **effective**) value of an alternating voltage or current is the value which would produce the same heat energy in a resistor as a direct voltage or current of the same magnitude. Since the r.m.s. value of a waveform is very much dependent upon its shape, values are only meaningful when dealing with a waveform of known shape. Where the shape of a waveform is...
Alternating voltage and current

\[ I_{\text{r.m.s.}} = 0.353 \times V_{\text{pk-pk}} = 0.353 \times 0.05 = 0.0177 \text{ A} \]
(or 17.7 mA).

**Example 4.6**

A sinusoidal voltage of 10 V pk–pk is applied to a resistor of 1 kΩ. What value of r.m.s. current will flow in the resistor?

**Solution**

This problem must be solved in two stages. First we will determine the peak–peak current in the resistor and then we shall convert this value into a corresponding r.m.s. quantity.

Since \[ I = \frac{V}{R} \] we can infer that \[ I_{\text{pk-pk}} = \frac{V_{\text{pk-pk}}}{R} \]

From which \[ I_{\text{pk-pk}} = \frac{10}{1000} = 0.01 = 10 \text{ mA pk–pk} \]

The required multiplying factor (peak–peak to r.m.s.) is 0.353. Thus:

\[ I_{\text{r.m.s.}} = 0.353 \times I_{\text{pk-pk}} = 0.353 \times 10 = 3.53 \text{ mA} \]

**Reactance**

When alternating voltages are applied to capacitors or inductors the magnitude of the current flowing will depend upon the value of capacitance or inductance and on the frequency of the voltage. In effect, capacitors and inductors oppose the flow of current in much the same way as a resistor. The important difference is that the effective resistance (or reactance) of the component varies with frequency (unlike the case of a resistor where the magnitude of the current does not change with frequency).

**Capacitive reactance**

The reactance of a capacitor is defined as the ratio of applied voltage to current and, like resistance, it is measured in ohms. The reactance of a capacitor is inversely proportional to both the value of capacitance and the frequency of the applied voltage. Capacitive reactance can be found by applying the following formula:
4. Alternating voltage and current

\[ X_c = \frac{1}{2\pi fC} \]

where \( X_c \) is the reactance (in ohms), \( f \) is the frequency (in hertz), and \( C \) is the capacitance (in farads).

Capacitive reactance falls as frequency increases, as shown in Fig. 4.5. The applied voltage, \( V_c \), and current, \( I_c \), flowing in a pure capacitive reactance will differ in phase by an angle of 90° or \( \pi/2 \) radians (the current leads the voltage). This relationship is illustrated in the current and voltage waveforms (drawn to a common time scale) shown in Fig. 4.6 and as a phasor diagram shown in Fig. 4.7.

**Example 4.7**

Determine the reactance of a 1 \( \mu \)F capacitor at (a) 100 Hz and (b) 10 kHz.

**Solution**

This problem is solved using the expression:

\[ X_c = \frac{1}{2\pi fC} \]

(a) At 100 Hz

\[ X_c = \frac{1}{2\pi \times 100 \times 1 \times 10^{-6}} = \frac{0.159}{10^{-4}} = 1.59 \times 10^2 \]

or

\[ X_c = 15.9 \text{ k}\Omega \]

(b) At 10 kHz

\[ X_c = \frac{1}{2\pi \times 1 \times 10^5 \times 1 \times 10^{-6}} = \frac{0.159}{10^{-2}} = 0.159 \times 10^2 \]

or

\[ X_c = 159 \Omega \]

**Example 4.8**

A 100 nF capacitor is to form part of a filter connected across a 240 V 50 Hz mains supply. What current will flow in the capacitor?

**Solution**

First we must find the reactance of the capacitor:

\[ X_c = \frac{1}{2\pi \times 50 \times 100 \times 10^{-9}} = 31.8 \times 10^3 = 31.8 \text{ k}\Omega \]

The r.m.s. current flowing in the capacitor will thus be:
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\[ I_c = \frac{V_c}{X_c} = \frac{240}{31.8 \times 10^3} = 7.5 \times 10^{-3} = 7.5 \text{ mA} \]

**Inductive reactance**

The reactance of an inductor is defined as the ratio of applied voltage to current and, like resistance, it is measured in ohms. The reactance of an inductor is directly proportional to both the value of inductance and the frequency of the applied voltage. Inductive reactance can be found by applying the formula:

\[ X_L = 2\pi fL \]

where \( X_L \) is the reactance in \( \Omega \), \( f \) is the frequency in Hz and \( L \) is the inductance in H.

Inductive reactance increases linearly with frequency as shown in Fig. 4.8. The applied voltage, \( V_L \), and current, \( I_L \), developed across a pure inductive reactance will differ in phase by an angle of 90° or \( \pi/2 \) radians (the current lags the voltage). This relationship is illustrated in the current and voltage waveforms (drawn to a common time scale) shown in Fig. 4.9 and as a phasor diagram shown in Fig. 4.10.

**Example 4.9**

Determine the reactance of a 10 mH inductor at (a) 100 Hz and (b) at 10 kHz.

(a) at 100 Hz

\[ X_L = 2\pi \times 100 \times 10 \times 10^{-3} = 6.28 \Omega \]

(b) At 10 kHz

\[ X_L = 2\pi \times 10 \times 10^3 \times 10 \times 10^{-3} = 628 \Omega \]

**Example 4.10**

A 100 mH inductor of negligible resistance is to form part of a filter which carries a current of 20 mA at 400 Hz. What voltage drop will be developed across the inductor?

**Solution**

The reactance of the inductor will be given by:

\[ X_L = 2\pi \times 400 \times 100 \times 10^{-3} = 251 \Omega \]

The r.m.s. voltage developed across the inductor will be given by:

\[ V_L = I_L \times X_L = 20 \text{ mA} \times 251 \Omega = 5.02 \text{ V} \]

In this example, it is important to note that we have assumed that the d.c. resistance of the
inductor is negligible by comparison with its reactance. Where this is not the case, it will be necessary to determine the **impedance** of the component and use this to determine the voltage drop.

**Impedance**

Fig. 4.11 shows two circuits which contain both resistance and reactance. These circuits are said to exhibit impedance (a combination of resistance and reactance) which, like resistance and reactance, is measured in ohms.

The impedance of the circuits shown in Fig. 4.11 is simply the ratio of supply voltage, \( V_s \), to supply current, \( I_s \). The impedance of the simple C–R and L–R circuits shown in Fig. 4.11 can be found by using the impedance triangle shown in Fig. 4.12. In either case, the impedance of the circuit is given by:

\[
Z = \sqrt{R^2 + X^2}
\]

and the phase angle (between \( V_s \) and \( I_s \)) is given by:

\[
\phi = \tan^{-1}\left(\frac{X}{R}\right)
\]

where \( Z \) is the impedance (in ohms), \( X \) is the reactance, either capacitive or inductive (expressed in ohms), \( R \) is the resistance (in ohms) and \( \phi \) is the phase angle in radians.

**Example 4.11**

A 2 \( \mu \)F capacitor is connected in series with a 100 \( \Omega \) resistor across a 115 V 400 Hz a.c. supply. Determine the impedance of the circuit and the current taken from the supply.

**Solution**

First we must find the reactance of the capacitor, \( X_C \):

\[
X_C = \frac{1}{2\pi f C} = \frac{1}{6.28 \times 400 \times 2 \times 10^{-6}} = \frac{10^6}{5024} = 199 \Omega
\]

Now we can find the impedance of the \( C–R \) series circuit:

\[
Z = \sqrt{R^2 + X^2} = \sqrt{199^2 + 100^2} = \sqrt{49601} = 223 \Omega
\]

The current taken from the supply can now be found:

\[
I_s = \frac{V_s}{Z} = \frac{115}{223} = 0.52 \text{ A}
\]

**Power factor**

The power factor in an a.c. circuit containing resistance and reactance is simply the ratio of true power to apparent power. Hence:

\[
\text{power factor} = \frac{\text{true power}}{\text{apparent power}}
\]

The **true power** in an a.c. circuit is the power which is actually dissipated in the resistive component. Thus:

\[
\text{true power} = I_s^2 \times R \quad \text{(watts)}
\]

The **apparent power** in an a.c. circuit is the power which is apparently consumed by the circuit and is the product of the supply current and supply voltage (note that this is not the same as
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the power which is actually dissipated as heat). Hence:

apparent power = \( I_S \times V_S \) (volt-amperes)

Hence

power factor = \( \frac{I_S^2 \times R}{I_S \times V_S} = \frac{I_S^2 \times R}{I_S \times (I_S \times Z)} = \frac{R}{Z} \)

From Fig. 4.12, \( \frac{R}{Z} = \cos \phi \)

Hence the power factor of a series a.c. circuit can be found from the cosine of the phase angle.

Example 4.12

A choke (a form of inductor) having an inductance of 150 mH and resistance of 250 \( \Omega \) is connected to a 115 V 400 Hz a.c. supply. Determine the power factor of the choke and the current taken from the supply.

Solution

First we must find the reactance of the inductor,

\[ X_L = 2\pi \times 400 \times 0.15 = 376.8 \Omega \]

We can now determine the power factor from:

\[ \text{power factor} = \frac{R}{Z} = \frac{250}{376.8} = 0.663 \]

The impedance of the choke, \( Z \), will be given by:

\[ Z = \sqrt{R^2 + X^2} = \sqrt{376.8^2 + 250^2} = 452 \Omega \]

Finally, the current taken from the supply will be:

\[ I_S = \frac{V_S}{Z} = \frac{115}{452} = 0.254 \text{ A} \]

L–C circuits

Two forms of L–C circuit are illustrated in Fig. 4.13. Fig. 4.13(a) is a series resonant circuit while Fig. 4.13(b) constitutes a parallel resonant circuit. The impedance of both circuits varies in a complex manner with frequency.

The impedance of the series circuit in Fig. 4.13(a) is given by:

\[ Z = \sqrt{X_L^2 - X_C^2} \]

where \( Z \) is the impedance of the circuit (in ohms), and \( X_L \) and \( X_C \) are the reactances of the inductor and capacitor, respectively (both expressed in ohms).

The phase angle (between the supply voltage and current) will be \(+\pi/2 \text{ rad} \) (i.e. \(+90^\circ \)) when \( X_L > X_C \) (above resonance) or \(-\pi/2 \text{ rad} \) (or \(-90^\circ \)) when \( X_C > X_L \) (below resonance).

At a particular frequency (known as the series resonant frequency) the reactance of the capacitor, \( X_C \), will be equal in magnitude (but of opposite sign) to that of the inductor, \( X_L \). Due to this effective cancellation of the reactance, the impedance of the series resonant circuit will be zero at resonance. The supply current will have a maximum value at resonance (infinite in the case of a perfect series resonant circuit supplied from an ideal voltage source!).

The impedance of the parallel circuit in Fig. 4.13 is given by:

\[ Z = \frac{X_L \times X_C}{\sqrt{X_L^2 - X_C^2}} \]

where \( Z \) is the impedance of the circuit (in \( \Omega \)), and \( X_L \) and \( X_C \) are the reactances of the inductor and capacitor, respectively (both expressed in \( \Omega \)).

The phase angle (between the supply voltage and current) will be \(+\pi/2 \text{ rad} \) (i.e. \(+90^\circ \)) when \( X_L > X_C \) (above resonance) or \(-\pi/2 \text{ rad} \) (or \(-90^\circ \)) when \( X_C > X_L \) (below resonance).

Figure 4.13 Series resonant and parallel resonant L–C and L–C–R circuits
At a particular frequency (known as the parallel resonant frequency) the reactance of the capacitor, $X_C$, will be equal in magnitude (but of opposite sign) to that of the inductor, $X_L$. At resonance, the denominator in the formula for impedance becomes zero and thus the circuit has an infinite impedance at resonance. The supply current will have a minimum value at resonance (zero in the case of a perfect parallel resonant circuit).

**L–C–R circuits**

Two forms of L–C–R network are illustrated in Figs 4.13; Fig. 4.13(c) is series resonant while Fig. 4.13(d) is parallel resonant. As in the case of their simpler L–C counterparts, the impedance of each circuit varies in a complex manner with frequency.

The impedance of the series circuit of Fig. 4.13(c) is given by:

$$ Z = \sqrt{R^2 + (X_L - X_C)^2} $$

where $Z$ is the impedance of the series circuit (in ohms), $R$ is the resistance (in $\Omega$), $X_L$ is the inductive reactance (in $\Omega$) and $X_C$ is the capacitive reactance (also in $\Omega$). At resonance the circuit has a minimum impedance (equal to $R$).

The phase angle (between the supply voltage and current) will be given by:

$$ \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) $$

The impedance of the parallel circuit of Fig. 4.13(d) is given by:

$$ Z = \frac{R \times X_L \times X_C}{\sqrt{(X_L^2 - X_C^2) + R^2(X_L - X_C)^2}} $$

where $Z$ is the impedance of the series circuit (in ohms), $R$ is the resistance (in $\Omega$), $X_L$ is the inductive reactance (in $\Omega$) and $X_C$ is the capacitive reactance (also in $\Omega$). At resonance the circuit has a minimum impedance (equal to $R$).

The phase angle (between the supply voltage and current) will be given by:

$$ \phi = \tan^{-1}\left(\frac{R(X_L - X_C)}{X_L \times X_C}\right) $$

**Resonance**

The frequency at which the impedance is minimum for a series resonant circuit or maximum in the case of a parallel resonant circuit is known as the resonant frequency. The resonant frequency is given by:

$$ f = \frac{1}{2\pi\sqrt{LC}} $$

where $f_0$ is the resonant frequency (in hertz), $L$ is the inductance (in henries) and $C$ is the capacitance (in farads).

Typical impedance-frequency characteristics for series and parallel tuned circuits are shown in Figs 4.14 and 4.15.

The series L–C–R tuned circuit has a minimum impedance at resonance (equal to $R$) and thus maximum current will flow. The circuit is consequently known as an acceptor circuit.

The parallel L–C–R tuned circuit has a maximum impedance at resonance (equal to $R$) and thus minimum current will flow. The circuit is consequently known as a rejector circuit.

**Quality factor**

The quality of a resonant (or tuned) circuit is measured by its $Q$-factor. The higher the $Q$-factor, the sharper the response (narrower bandwidth), conversely the lower the $Q$-factor, the flatter the response (wider bandwidth), see Fig. 4.16. In the case of the series tuned circuit, the $Q$-factor...
4. Alternating voltage and current

![Figure 4.15 Impedance versus frequency for a parallel L–C–R rejector circuit](image)

The relationship between bandwidth and Q-factor is:

$$\text{Bandwidth} = f_2 - f_1 = \frac{f_0}{Q}$$

$$Q = \frac{2\pi f_0 L}{R}$$

where $f_2$ and $f_1$ are respectively the upper and lower cut-off (or half-power) frequencies (in hertz), $f_0$ is the resonant frequency (in hertz), and $Q$ is the Q-factor.

Example 4.13

A parallel L–C circuit is to be resonant at a frequency of 400 Hz. If a 100 mH inductor is available, determine the value of capacitance required.

**Solution**

Re-arranging the formula

$$f = \frac{1}{2\pi\sqrt{LC}}$$

to make $C$ the subject gives:

$$C = \frac{1}{f_0^2 (2\pi)^2 L}$$

Thus

$$C = \frac{1}{400^2 \times 39.4 \times 100 \times 10^{-3}} = 1.58 \times 10^{-6} = 1.58 \mu F$$

This value can be made from preferred values using a 2.2 μF capacitor connected in series with a 5.6 μF capacitor.

Example 4.14

A series L–C–R circuit comprises an inductor of 20 mH, a capacitor of 10 nF, and a resistor of 100 Ω. If the circuit is supplied with a sinusoidal signal of 1.5 V at a frequency of 2 kHz, determine the current supplied and the voltage developed across the resistor.
Alternating voltage and current

Solution

First we need to determine the values of inductive reactance, $X_L$, and capacitive reactance $X_C$:

$$X_L = 2\pi f L = 6.28 \times 2 \times 10^3 \times 20 \times 10^{-3} = 251 \, \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{6.28 \times 2 \times 10^3 \times 100 \times 10^{-9}} = 796.2 \, \Omega$$

The impedance of the series circuit can now be calculated:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{100^2 + (251.2 - 796.2)^2}$$

From which:

$$Z = \sqrt{10,000 + 297,025} = \sqrt{307,025} = 554 \, \Omega$$

The current flowing in the series circuit will be given by:

$$I = \frac{\sqrt{1.5}}{554} = 0.0027 = 2.7 \, \text{mA}$$

The voltage developed across the resistor can now be calculated using:

$$V = IR = 2.7 \, \text{mA} \times 100 \, \Omega = 270 \, \text{mV}$$

Transformers

Transformers provide us with a means of coupling a.c. power or signals from one circuit to another. Voltage may be stepped-up (secondary voltage greater than primary voltage) or stepped-down (secondary voltage less than primary voltage). Since no increase in power is possible (transformers are passive components like resistors, capacitors and inductors) an increase in secondary voltage can only be achieved at the expense of a corresponding reduction in secondary current, and vice versa (in fact, the secondary power will be very slightly less than the primary power due to losses within the transformer). Typical applications for transformers include stepping-up or stepping-down mains voltages in power supplies, coupling signals in AF amplifiers to achieve impedance matching and to isolate d.c. potentials associated with active components.

The electrical characteristics of a transformer are determined by a number of factors including the core material and physical dimensions. The specifications for a transformer usually include the rated primary and secondary voltages and currents, the required power rating (i.e. the maximum power, usually expressed in volt-amperes, VA) which can be continuously delivered by the transformer under a given set of conditions, the frequency range for the

![Figure 4.18 A selection of transformers with power ratings from 0.1 VA to 100 VA](image.png)

<table>
<thead>
<tr>
<th>Property</th>
<th>Air cored</th>
<th>Ferrite cored</th>
<th>Iron cored (audio)</th>
<th>Iron cored (power)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core material/construction</td>
<td>Air</td>
<td>Ferrite ring or pot</td>
<td>Laminated steel</td>
<td>Laminated steel</td>
</tr>
<tr>
<td>Typical frequency range (Hz)</td>
<td>30 M to 1 G</td>
<td>10 k to 10 M</td>
<td>20 to 20 k</td>
<td>50 to 400</td>
</tr>
<tr>
<td>Typical power rating (VA)</td>
<td>(see note)</td>
<td>1 to 200</td>
<td>0.1 to 50</td>
<td>3 to 500</td>
</tr>
<tr>
<td>Typical regulation</td>
<td>(see note)</td>
<td>(see note)</td>
<td>(see note)</td>
<td>5% to 15%</td>
</tr>
<tr>
<td>Typical applications</td>
<td>RF tuned circuits and filters</td>
<td>Filters and HF transformers, switched mode power supplies</td>
<td>Smoothing chokes and filters, audio matching</td>
<td>Power supplies</td>
</tr>
</tbody>
</table>

Note: Not usually important for this type of transformer
4. Alternating voltage and current

Voltage and turns ratio

The principle of the transformer is illustrated in Fig. 4.21. The primary and secondary windings are wound on a common low-reluctance magnetic core. The alternating flux generated by the primary winding is therefore coupled into the secondary winding (very little flux escapes due to leakage). A sinusoidal current flowing in the primary winding produces a sinusoidal flux. At any instant the flux in the transformer is given by the equation:

\[ \phi = \phi_{\text{max}} \sin(2\pi ft) \]

where \( \phi_{\text{max}} \) is the maximum value of flux (in webers), \( f \) is the frequency of the applied current (in hertz), and \( t \) is the time (in seconds).

The r.m.s. value of the primary voltage, \( V_p \), is given by:

\[ V_p = \frac{4.44fN_p\phi_{\text{max}}}{p} \]

Similarly, the r.m.s. value of the secondary voltage, \( V_s \), is given by:

\[ V_s = \frac{4.44fN_s\phi_{\text{max}}}{p} \]

component (usually stated as upper and lower working frequency limits) and the regulation of a transformer (usually expressed as a percentage of full-load). This last specification is a measure of the ability of a transformer to maintain its rated output voltage under load.

Table 4.2 summarizes the properties of three common types of transformer. Figure 4.20 shows the construction of a typical iron-cored power transformer.

Figure 4.21 The transformer principle
Now
\[ \frac{V_P}{V_S} = \frac{N_P}{N_S} \]

where \( \frac{N_P}{N_S} \) is the turns ratio of the transformer.

Assuming that the transformer is loss-free, primary and secondary powers \( (P_P \text{ and } P_S) \) respectively will be identical. Hence:
\[ P_P = P_S \text{ thus } V_P \times I_P = V_S \times I_S \]

Hence
\[ \frac{V_P}{V_S} = \frac{I_S}{I_P} \text{ and } \frac{I_S}{I_P} = \frac{N_P}{N_S} \]

Finally, it is sometimes convenient to refer to a turns-per-volt rating for a transformer. This rating is given by:
\[ \text{turns-per-volt} = \frac{N_P}{V_P} = \frac{N_S}{V_S} \]

**Example 4.15**

A transformer has 2,000 primary turns and 120 secondary turns. If the primary is connected to a 220 V r.m.s. a.c. mains supply, determine the secondary voltage.

**Solution**

Re-arranging \( \frac{V_P}{V_S} = \frac{N_P}{N_S} \) gives:
\[ V_S = \frac{N_S \times V_P}{N_P} = \frac{120 \times 220}{2,000} = 13.2 \text{ V} \]

**Example 4.16**

A transformer has 1,200 primary turns and is designed to operate with a 200 V a.c. supply. If the transformer is required to produce an output of 10 V, determine the number of secondary turns required. Assuming that the transformer is loss-free, determine the input (primary) current for a load current of 2.5 A.

**Solution**

Re-arranging \( \frac{V_P}{V_S} = \frac{N_P}{N_S} \) gives:

*Figure 4.22* Resonant air-cored transformer arrangement. The two inductors are tuned to resonance at the operating frequency (145 MHz) by means of the two small preset capacitors.

*Figure 4.23* This small 1:1 ratio toroidal transformer forms part of a noise filter connected in the input circuit of a switched mode power supply. The transformer is wound on a ferrite core and acts as a choke, reducing the high-frequency noise that would otherwise be radiated from the mains supply wiring.
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\[
N_s = \frac{N_p \times V_s}{V_p} = \frac{1,200 \times 10}{200} = 60 \text{ turns}
\]

Re-arranging \(\frac{I_s}{I_p} = \frac{N_p}{N_s}\) gives:

\[
N_s = \frac{N_s \times I_s}{N_p} = \frac{200 \times 2.5}{1,200} = 0.42 \text{ A}
\]

Practical investigation

Objective
To investigate reactance in an a.c. circuit.

Components and test equipment
Breadboard, digital or analogue meters with a.c. voltage and current ranges, sine wave signal generator (with an output impedance of 50 \(\Omega\) or less), 1 \(\mu\)F capacitor, 60 mH inductor (with low series loss resistance), test leads, connecting wire.

Procedure
Connect the circuit shown in Fig. 4.24 (capacitive reactance). Set the voltmeter and ammeter respectively to the 2 V and 20 mA ranges. Set the signal generator to produce a sine wave output at 100 Hz.

Adjust the signal generator output voltage so that the voltmeter reads exactly 1 V before measuring and recording the current (this should be less than 1 mA). Repeat this measurement at frequencies from 200 Hz to 1 kHz in steps of 100 Hz. At each step, check that the voltage is exactly 1 V (adjust if necessary).

Connect the circuit shown in Fig. 4.25 (inductive reactance). As before, set the voltmeter and ammeter to the 2 V and 20 mA ranges and set the signal generator to produce a sine wave output at 100 Hz.

Adjust the signal generator output for a voltage of exactly 1 V before measuring and recording the current. Repeat this measurement at frequencies from 200 Hz to 1 kHz in steps of 100 Hz. At each step, check that the voltage is exactly 1 V (adjust the signal generator output if necessary).

Connect the circuit shown in Fig. 4.26 (resonant circuit). As before, set the voltmeter and ammeter respectively to the 2 V and 20 mA ranges. Set the signal generator to produce a sine wave output at 100 Hz.

Adjust the signal generator output voltage so that the voltmeter reads exactly 1 V before measuring and recording the current. Repeat this measurement at frequencies from 200 Hz to 1 kHz in steps of 100 Hz. At each step, check that the voltage is exactly 1 V (adjust if necessary). Note the frequency at which the current takes a minimum value.
Measurements and calculations

Record your results in a table showing values of \( I_C, I_L \) and \( I_S \) at each frequency from 100 Hz to 1 kHz. Plot graphs showing how the current in each circuit varies over the frequency range 100 Hz to 1 kHz using the graph layout shown in Fig. 4.27. Calculate the resonant frequency of the \( L-C \) circuit shown in Fig. 4.26.

Conclusion

Comment on the shape of each graph. Is this what you would expect (recall that the current flowing in the circuit will be proportional to the reciprocal of the reactance)? Compare the measured resonant frequency with the calculated value. If they are not the same, suggest reasons for any difference.

![Figure 4.27 Graph layout for plotting the results](image-url)

Important formulae introduced in this chapter

Sine wave voltage:
(page 73)
\[ v = V_{\text{rms}} \sin (2\pi ft) \]

Sine wave voltage superimposed on a d.c. level:
(page 73)
\[ v = V_{\text{dc}} + V_{\text{rms}} \sin (2\pi ft) \]

Frequency and periodic time:
(page 73)
\[ t = \frac{1}{f} \text{ or } f = \frac{1}{t} \]

Peak and r.m.s. values for a sine wave:
(page 74)
\[ V_{\text{pk}} = 1.414 \times V_{\text{r.m.s}} \]
\[ V_{\text{r.m.s}} = 0.707 \times V_{\text{pk}} \]

Capacitive reactance:
(page 75)
\[ X_C = \frac{1}{2\pi f C} \]

Inductive reactance:
(page 76)
\[ X_L = 2\pi f L \]

Impedance of \( C-R \) or \( L-R \) in series:
(page 77)
\[ Z = \sqrt{R^2 + X^2} \]

Phase angle for \( C-R \) or \( L-R \) in series:
(page 77)
\[ \phi = \tan^{-1}\left(\frac{X}{R}\right) \]

Power factor:
(pages 77 and 78)
\[ \text{power factor} = \frac{\text{true power}}{\text{apparent power}} \]
\[ \text{power factor} = \frac{R}{Z} = \cos \phi \]

Resonant frequency of a tuned circuit:
(page 79)
\[ f = \frac{1}{2\pi \sqrt{LC}} \]
4. Alternating voltage and current

Bandwidth of a tuned circuit:
(page 80)

\[ \text{Bandwidth} = f_2 - f_1 = \frac{f_0}{Q} \]

Q-factor for a series tuned circuit:
(page 80)

\[ Q = \frac{2\pi f_s L}{R} \]

Flux in a transformer:
(page 82)

\[ \phi = \phi_{\text{max}} \sin(2\pi ft) \]

Transformer voltages:
(page 82)

\[ V_p = 4.44fN_p\phi_{\text{max}} \]
\[ V_s = 4.44fN_s\phi_{\text{max}} \]

Voltage and turns ratio:
(page 83)

\[ \frac{V_p}{V_s} = \frac{N_p}{N_s} \]

Current and turns ratio:
(page 83)

\[ \frac{I_s}{I_p} = \frac{N_p}{N_s} \]

Turns-per-volt:
(page 83)

\[ \text{turns-per-volt} = \frac{N_p}{V_p} = \frac{N_s}{V_s} \]

---

Symbols introduced in this chapter

Air-core transformer
Ferrite-core transformer
Iron/steel-core transformer
Multi-secondary transformer
Autotransformer
Tapped transformer

Figure 4.28 Circuit symbols introduced in this chapter

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Problems

4.1 A sine wave has a frequency of 250 Hz and an amplitude of 50 V. Determine its periodic time and r.m.s. value.

4.2 A sinusoidal voltage has an r.m.s. value of 240 V and a period of 16.7 ms. What is the frequency and peak value of the voltage?

4.3 Determine the frequency and peak–peak values of each of the waveforms shown in Fig. 4.29.

4.4 A sine wave has a frequency of 100 Hz and an amplitude of 20 V. Determine the instantaneous value of voltage (a) 2 ms and (b) 9 ms from the start of a cycle.
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4.10 A 10 uF capacitor is connected in series with a 500 Ω resistor across a 110 V 50 Hz a.c. supply. Determine the impedance of the circuit and the current taken from the supply.

4.11 A choke having an inductance of 1 H and resistance of 250 Ω is connected to a 220 V 60 Hz a.c. supply. Determine the power factor of the choke and the current taken from the supply.

4.12 A series-tuned L–C network is to be resonant at a frequency of 1.8 kHz. If a 60 mH inductor is available, determine the value of capacitance required.

4.13 Determine the impedance at 1 kHz of each of the circuits shown in Fig. 4.30.

4.14 A parallel resonant circuit employs a fixed inductor of 22 μH and a variable tuning capacitor. If the maximum and minimum values of capacitance are respectively 20 pF and 365 pF, determine the effective tuning range for the circuit.

4.15 A series L-C-R circuit comprises an inductor of 15 mH (with negligible resistance), a capacitor of 220 nF and a resistor of 100 Ω. If the circuit is supplied with a sinusoidal signal of 15 V at a frequency of 2 kHz, determine the current supplied and the voltage developed across the capacitor.

4.16 A 470 μH inductor has a resistance of 20 Ω. If the inductor is connected in series with a capacitor of 680 pF, determine the resonant frequency, Q-factor and bandwidth of the circuit.

4.5 A sinusoidal current of 20 mA pk–pk flows in a resistor of 1.5 kΩ. Determine the r.m.s. voltage applied.

4.6 Determine the reactance of a 220 nF capacitor at (a) 20 Hz and (b) 5 kHz.

4.7 A 47 nF capacitor is connected across the 240 V 50 Hz mains supply. Determine the r.m.s. current flowing in the capacitor.

4.8 Determine the reactance of a 33 mH inductor at (a) 50 Hz and (b) 7 kHz.

4.9 A 10 mH inductor of negligible resistance is used to form part of a filter connected in series with a 50 Hz mains supply. What voltage drop will appear across the inductor when a current of 1.5 A is flowing?

Figure 4.30 See Question 4.13
4. Alternating voltage and current

4.20 A transformer has 1,600 primary turns and 120 secondary turns. If the primary is connected to a 240 V r.m.s. a.c. mains supply, determine the secondary voltage.

4.21 A transformer has 800 primary turns and 60 secondary turns. If the secondary is connected to a load resistance of 15 Ω, determine the value of primary voltage required to produce a power of 22.5 W in the load (assume that the transformer is loss-free).

4.22 A transformer has 440 primary turns and operates from a 110 V a.c. supply. The transformer has two secondary windings each rated at 12 V 20 VA. Determine:
   (a) the turns-per-volt rating
   (b) the secondary turns (each winding)
   (c) the secondary current (each winding)
   (d) the full-load primary current.

Answers to these problems appear on pages 416 and 417.
Semiconductors

Chapter summary
This chapter introduces devices that are made from materials that are neither conductors nor insulators. These semiconductor materials form the basis of diodes, thyristors, triacs, transistors and integrated circuits. We start this chapter with a brief introduction to the principles of semiconductors before going on to examine the characteristics of each of the most common types of semiconductor.
5. Semiconductors

In Chapter 1 we described the simplified structure of an atom and showed that it contains both negative charge carriers (electrons) and positive charge carriers (protons). Electrons each carry a single unit of negative electric charge while protons each exhibit a single unit of positive charge. Since atoms normally contain an equal number of electrons and protons, the net charge present will be zero. For example, if an atom has 11 electrons, it will also contain 11 protons. The end result is that the negative charge of the electrons will be exactly balanced by the positive charge of the protons.

Electrons are in constant motion as they orbit around the nucleus of the atom. Electron orbits are organized into shells. The maximum number of electrons present in the first shell is 2, in the second shell 8, and in the third, fourth and fifth shells it is 18, 32 and 50, respectively. In electronics only the electron shell furthest from the nucleus of an atom is important. It is important to note that the movement of electrons only involves those present in the outer valence shell.

If the valence shell contains the maximum number of electrons possible the electrons are rigidly bonded together and the material has the properties of an insulator. If, however, the valence shell does not have its full complement of electrons, the electrons can be easily loosened from their orbital bonds, and the material has the properties associated with an electrical conductor.

An isolated silicon atom contains four electrons in its valence shell. When silicon atoms combine to form a solid crystal, each atom positions itself between four other silicon atoms in such a way that the valence shells overlap from one atom to another. This causes each individual valence electron to be shared by two atoms, as shown in Fig. 5.1. By sharing the electrons between four adjacent atoms each individual silicon atom appears to have eight electrons in its valence shell. This sharing of valence electrons is called covalent bonding.

In its pure state, silicon is an insulator because the covalent bonding rigidly holds all of the electrons, leaving no free (easily loosened) electrons to conduct current. If, however, an atom of a different element (i.e. an impurity) is introduced that has five electrons in its valence shell, a surplus electron will be present, as shown in Fig. 5.2. These free electrons become available for use as charge carriers and they can be made to move through the lattice by applying an external potential difference to the material.

Similarly, if the impurity element introduced into the pure silicon lattice has three electrons in its valence shell, the absence of the fourth electron needed for proper covalent bonding will produce a number of gaps into which electrons can fit, as shown in Fig. 5.3. These gaps are referred to as holes. Once again, current will flow when an external potential difference is applied to the material.
Regardless of whether the impurity element produces surplus electrons or holes, the material will no longer behave as an insulator, neither will it have the properties that we normally associate with a metallic conductor. Instead, we call the material a semiconductor – the term simply indicates that the substance is no longer a good insulator or a good conductor but is somewhere in between!

The process of introducing an atom of another (impurity) element into the lattice of an otherwise pure material is called doping. When the pure material has been doped with an impurity with five electrons in its valence shell (i.e. a pentavalent impurity) it will become an N-type material. If, however, the pure material is doped with an impurity having three electrons in its valence shell (i.e. a trivalent impurity) it will become a P-type material. N-type semiconductor material contains an excess of negative charge carriers, and P-type material contains an excess of positive charge carriers.

**Semiconductor diodes**

When a junction is formed between N-type and P-type semiconductor materials, the resulting device is called a diode. This component offers an extremely low resistance to current flow in one direction and an extremely high resistance to current flow in the other. This characteristic allows the diode to be used in applications that require a circuit to behave differently according to the direction of current flowing in it.

An ideal diode would pass an infinite current in one direction and no current at all in the other direction. In addition, the diode would start to conduct current when the smallest of voltages was present. In practice, a small voltage must be applied before conduction takes place. Furthermore, a small leakage current will flow in the reverse direction. This leakage current is usually a very small fraction of the current that flows in the forward direction.

If the P-type semiconductor material is made positive relative to the N-type material by an amount greater than its forward threshold voltage (about 0.6 V if the material is silicon and 0.2 V if the material is germanium), the diode will freely pass current. If, on the other hand, the P-type material is made negative relative to the N-type material, virtually no current will flow unless the applied voltage exceeds the maximum (breakdown) voltage that the device can withstand. Note that a normal diode will be destroyed if its reverse breakdown voltage is exceeded.

A semiconductor junction diode is shown in Fig. 5.4. The connection to the P-type material is referred to as the anode while that to the N-type material is called the cathode. With no externally applied potential, electrons from the N-type material will cross into the P-type region and fill some of the vacant holes. This action will result in the production of a region either side of the junction in which there are no free charge carriers. This zone is known as the depletion region.

![Figure 5.4 A P–N junction diode](image-url)
5. Semiconductors

Fig. 5.5(a) shows a junction diode in which the anode is made positive with respect to the cathode. In this **forward-biased** condition, the diode freely passes current. Figure 5.5(b) shows a diode with the cathode made positive with respect to the cathode. In this **reverse-biased** condition, the diode passes a negligible amount of current. In the freely conducting forward-biased state, the diode acts rather like a closed switch. In the reverse-biased state, the diode acts like an open switch.

If a positive voltage is applied to the P-type material, the free positive charge carriers will be repelled and they will move away from the positive potential towards the junction. Likewise, the negative potential applied to the N-type material will cause the free negative charge carriers to move away from the negative potential towards the junction.

When the positive and negative charge carriers arrive at the junction, they will attract one another and combine (recall that unlike charges attract). As each negative and positive charge carrier combine at the junction, a new negative and positive charge carrier will be introduced to the semiconductor material from the voltage source. As these new charge carriers enter the semiconductor material, they will move towards the junction and combine. Thus, current flow is established and it will continue for as long as the voltage is applied.

As stated earlier, the **forward threshold voltage** must be exceeded before the diode will conduct. The forward threshold voltage must be high enough to completely remove the depletion layer and force charge carriers to move across the junction. With silicon diodes, this forward threshold voltage is approximately 0.6 V to 0.7 V. With germanium diodes, the forward threshold voltage is approximately 0.2 V to 0.3 V.

Fig. 5.6 shows typical characteristics for small germanium and silicon diodes. It is worth noting...
that diodes are limited by the amount of forward current and reverse voltage they can withstand. This limit is based on the physical size and construction of the diode.

In the case of a reverse-biased diode, the P-type material is negatively biased relative to the N-type material. In this case, the negative potential applied to the P-type material attracts the positive charge carriers, drawing them away from the junction. Likewise, the positive potential applied to the N-type material attracts the negative charge carriers away from the junction. This leaves the junction area depleted; virtually no charge carriers exist. Therefore, the junction area becomes an insulator, and current flow is inhibited. The reverse bias potential may be increased to the reverse breakdown voltage for which the particular diode is rated. As in the case of the maximum forward current rating, the reverse breakdown voltage is specified by the manufacturer. The reverse breakdown voltage is usually very much higher than the forward threshold voltage. A typical general-purpose diode may be specified as having a forward threshold voltage of 0.6 V and a reverse breakdown voltage of 200 V. If the latter is exceeded, the diode may suffer irreversible damage. It is also worth noting that, where diodes are designed for use as rectifiers, manufacturers often quote **peak inverse voltage (PIV)** or **maximum reverse repetitive voltage** ($V_{RRM}$) rather than maximum reverse breakdown voltage.

Fig. 5.7 shows a test circuit for obtaining diode characteristics (note that the diode must be reverse connected in order to obtain the reverse characteristic).

**Example 5.1**
The characteristic shown in Fig. 5.8 refers to a germanium diode. Determine the resistance of the diode when (a) the forward current is 2.5 mA and (b) when the forward voltage is 0.65 V.

**Solution**

(a) When $I_F = 2.5$ mA the corresponding value of $V_F$ can be read from the graph. This shows that $V_F = 0.43$ V. The resistance of the diode at this point on the characteristic will be given by:

$$R = \frac{V_F}{I_F} = \frac{0.43 \text{ V}}{2.5 \text{ mA}} = 172 \Omega$$

(b) When $V_F = 0.65$ V the corresponding value of $I_F$ can be read from the graph. This shows that $I_F = 7.4$ mA. The resistance of the diode at this point on the characteristic will be given by:

$$R = \frac{V_F}{I_F} = \frac{0.65 \text{ V}}{7.4 \text{ mA}} = 88 \Omega$$

This example shows how the resistance of a diode does not remain constant but instead changes according to the point on the characteristic at which it is operating.
5. Semiconductors

**Diode types**

Diodes are often divided into **signal** or **rectifier** types according to their principal field of application. Signal diodes require consistent forward characteristics with low forward voltage drop. Rectifier diodes need to be able to cope with high values of reverse voltage and large values of forward current, consistency of characteristics is of secondary importance in such applications. Table 5.1 summarizes the characteristics of some common semiconductor diodes while a selection of diodes are shown in Fig. 5.9.

**Zener diodes**

Zener diodes are heavily doped silicon diodes which, unlike normal diodes, exhibit an abrupt reverse breakdown at relatively low voltages (typically less than 6 V). A similar effect occurs in less heavily doped diodes. These **avalanche diodes** also exhibit a rapid breakdown with negligible current flowing below the avalanche voltage and an increasingly large current flowing once the avalanche voltage has been reached. For avalanche diodes, this breakdown voltage usually occurs at voltages above 6 V. In practice, however, both types of diode are referred to as zener diodes. A typical characteristic for a 12 V zener diode is shown in Fig. 5.10.

Whereas reverse breakdown is a highly undesirable effect in circuits that use conventional diodes, it can be extremely useful in the case of zener diodes where the breakdown voltage is precisely known. When a diode is undergoing reverse breakdown and provided its maximum

---

**Table 5.1 Characteristics of some common semiconductor diodes**

<table>
<thead>
<tr>
<th>Device</th>
<th>Material</th>
<th>PIV</th>
<th>$I_F$ max.</th>
<th>$I_R$ max.</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N4148</td>
<td>Silicon</td>
<td>100 V</td>
<td>76 mA</td>
<td>25 nA</td>
<td>General purpose</td>
</tr>
<tr>
<td>1N914</td>
<td>Silicon</td>
<td>100 V</td>
<td>75 mA</td>
<td>25 nA</td>
<td>General purpose</td>
</tr>
<tr>
<td>AA113</td>
<td>Germanium</td>
<td>60 V</td>
<td>10 mA</td>
<td>200 μA</td>
<td>RF detector</td>
</tr>
<tr>
<td>OA47</td>
<td>Germanium</td>
<td>25 V</td>
<td>110 mA</td>
<td>100 μA</td>
<td>Signal detector</td>
</tr>
<tr>
<td>OA91</td>
<td>Germanium</td>
<td>115 V</td>
<td>50 mA</td>
<td>275 μA</td>
<td>General purpose</td>
</tr>
<tr>
<td>1N4001</td>
<td>Silicon</td>
<td>50 V</td>
<td>1 A</td>
<td>10 μA</td>
<td>Low-voltage rectifier</td>
</tr>
<tr>
<td>1N5404</td>
<td>Silicon</td>
<td>400 V</td>
<td>3 A</td>
<td>10 μA</td>
<td>High-voltage rectifier</td>
</tr>
<tr>
<td>BY127</td>
<td>Silicon</td>
<td>1,250 V</td>
<td>1 A</td>
<td>10 μA</td>
<td>High-voltage rectifier</td>
</tr>
</tbody>
</table>

---

**Figure 5.9** A selection of diodes including power and bridge rectifiers, thyristors, signal and zener diodes

**Figure 5.10** Typical characteristic for a 12 V zener diode
which, in turn, varies with the reverse voltage applied to the diode. This allows a diode to be used as a voltage controlled capacitor. Diodes that are specially manufactured to make use of this effect (and which produce comparatively large changes in capacitance for a small change in reverse voltage) are known as variable capacitance diodes (or varicaps). Such diodes are used (often in pairs) to provide tuning in radio and TV receivers. A typical characteristic for a variable capacitance diode is shown in Fig. 5.12. Table 5.3 summarizes the characteristics of several common variable capacitance diodes.

**Thyristors**

Thyristors (or silicon controlled rectifiers) are three-terminal devices which can be used for switching and a.c. power control. Thyristors can switch very rapidly from a conducting to a non-conducting state. In the off state, the thyristor exhibits negligible leakage current, while in the on state the device exhibits very low resistance. This

---

**Variable capacitance diodes**

The capacitance of a reverse-biased diode junction will depend on the width of the depletion layer

![Figure 5.11 Zener diode test circuit](image)

![Figure 5.12 Typical characteristic for a variable capacitance diode](image)

**Table 5.2 Characteristics of some common zener diodes**

<table>
<thead>
<tr>
<th>Series</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BZY88</td>
<td>Miniature glass encapsulated diodes rated at 500 mW (at 25°C). Zener voltages range from 2.7 V to 15 V (voltages are quoted for 5 mA reverse current at 25°C)</td>
</tr>
<tr>
<td>BZX61</td>
<td>Encapsulated alloy junction rated at 1.3 W (25°C ambient). Zener voltages range from 7.5 V to 72 V</td>
</tr>
<tr>
<td>BZX85</td>
<td>Medium-power glass-encapsulated diodes rated at 1.3 W and offering zener voltages in the range 5.1 V to 62 V</td>
</tr>
<tr>
<td>BZY93</td>
<td>High-power diodes in stud mounting encapsulation. Rated at 20 W for ambient temperatures up to 75°C. Zener voltages range from 9.1 V to 75 V</td>
</tr>
<tr>
<td>1N5333</td>
<td>Plastic encapsulated diodes rated at 5 W. Zener voltages range from 3.3 V to 24 V</td>
</tr>
</tbody>
</table>
5. Semiconductors

Table 5.3 Characteristics of several common types of variable capacitance diode

<table>
<thead>
<tr>
<th>Type</th>
<th>Capacitance (at 4V)</th>
<th>Capacitance ratio</th>
<th>Q-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N5450</td>
<td>33 pF</td>
<td>2.6 (4 V to 60 V)</td>
<td>350</td>
</tr>
<tr>
<td>MV1404</td>
<td>50 pF</td>
<td>&gt;10 (2 V to 10 V)</td>
<td>200</td>
</tr>
<tr>
<td>MV2103</td>
<td>10 pF</td>
<td>2 (4 V to 60 V)</td>
<td>400</td>
</tr>
<tr>
<td>MV2115</td>
<td>100 pF</td>
<td>2.6 (4 V to 60 V)</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5.4 Characteristics of several common types of thyristor

<table>
<thead>
<tr>
<th>Type</th>
<th>I$_{\text{ave}}$ (A)</th>
<th>V$_{\text{RMM}}$ (V)</th>
<th>V$_{\text{GT}}$ (V)</th>
<th>I$_{\text{GT}}$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2N4444</td>
<td>5.1</td>
<td>600</td>
<td>1.5</td>
<td>30</td>
</tr>
<tr>
<td>BT106</td>
<td>1</td>
<td>700</td>
<td>3.5</td>
<td>50</td>
</tr>
<tr>
<td>BT152</td>
<td>13</td>
<td>600</td>
<td>1</td>
<td>32</td>
</tr>
<tr>
<td>BTY79-400R</td>
<td>6.4</td>
<td>400</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>TIC106D</td>
<td>3.2</td>
<td>400</td>
<td>1.2</td>
<td>0.2</td>
</tr>
<tr>
<td>TIC126D</td>
<td>7.5</td>
<td>400</td>
<td>2.5</td>
<td>20</td>
</tr>
</tbody>
</table>

results in very little power loss within the thyristor even when appreciable power levels are being controlled. Once switched into the conducting state, the thyristor will remain conducting (i.e. it is latched in the on state) until the forward current is removed from the device. In d.c. applications this necessitates the interruption (or disconnection) of the supply before the device can be reset into its non-conducting state. Where the device is used with an alternating supply, the device will automatically become reset whenever the main supply reverses. The device can then be triggered on the next half-cycle having correct polarity to permit conduction. Like their conventional silicon diode counterparts, thyristors have anode and cathode connections; control is applied by means of a gate terminal (see Fig. 5.13). The device is triggered into the conducting (on state) by means of the application of a current pulse to this terminal. The effective triggering of a thyristor requires a gate trigger pulse having a fast rise time derived from a low-resistance source. Triggering can become erratic when insufficient gate current is available or when the gate current changes slowly. Table 5.4 summarizes the characteristics of several common thyristors.

Table 5.5 summarizes results in very little power loss within the thyristor even when appreciable power levels are being controlled. Once switched into the conducting state, the thyristor will remain conducting (i.e. it is latched in the on state) until the forward current is removed from the device. In d.c. applications this necessitates the interruption (or disconnection) of the supply before the device can be reset into its non-conducting state. Where the device is used with an alternating supply, the device will automatically become reset whenever the main supply reverses. The device can then be triggered on the next half-cycle having correct polarity to permit conduction. Like their conventional silicon diode counterparts, thyristors have anode and cathode connections; control is applied by means of a gate terminal (see Fig. 5.13). The device is triggered into the conducting (on state) by means of the application of a current pulse to this terminal. The effective triggering of a thyristor requires a gate trigger pulse having a fast rise time derived from a low-resistance source. Triggering can become erratic when insufficient gate current is available or when the gate current changes slowly. Table 5.4 summarizes the characteristics of several common thyristors.

**Triacs**

Triacs are a refinement of the thyristor which, when triggered, conduct on both positive and negative half-cycles of the applied voltage. Triacs have three terminals known as main-terminal one (MT1), main-terminal two (MT2) and gate (G), as shown in Fig. 5.14. Triacs can be triggered by both positive and negative voltages applied between G and MT1 with positive and negative voltages present at MT2, respectively. Triacs thus provide full-wave control and offer superior performance in a.c. power control applications when compared with thyristors which only provide half-wave control. Table 5.5 summarizes
In order to limit the forward current of an LED to an appropriate value, it is usually necessary to include a fixed resistor in series with an LED indicator, as shown in Fig. 5.16. The value of the resistor may be calculated from:

$$R = \frac{V - V_F}{I}$$

where $V_F$ is the forward voltage drop produced by the LED and $V$ is the applied voltage. Note that it is usually safe to assume that $V_F$ will be 2 V and choose the nearest preferred value for $R$.

**Example 5.2**

An LED is to be used to indicate the presence of a 21 V d.c. supply rail. If the LED has a nominal forward voltage of 2.2 V, and is rated at a current of 15 mA, determine the value of series resistor required.

**Solution**

Here we can use the formula:

$$R = \frac{21\text{ V} - 2.2\text{ V}}{15\text{ mA}} = \frac{18.8\text{ V}}{15\text{ mA}} = 1.25\text{ k}\Omega$$
5. Semiconductors

**Table 5.6** Characteristics of some common types of LED

<table>
<thead>
<tr>
<th>Resistance (Ω)</th>
<th>LED type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Miniature</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>3</td>
</tr>
<tr>
<td>Maximum forward current (mA)</td>
<td>40</td>
</tr>
<tr>
<td>Typical forward current (mA)</td>
<td>12</td>
</tr>
<tr>
<td>Typical forward voltage drop (V)</td>
<td>2.1</td>
</tr>
<tr>
<td>Maximum reverse voltage (V)</td>
<td>5</td>
</tr>
<tr>
<td>Maximum power dissipation (mW)</td>
<td>150</td>
</tr>
<tr>
<td>Peak wavelength (nm)</td>
<td>690</td>
</tr>
</tbody>
</table>

The nearest preferred value is 1.2 kΩ. The power dissipated in the resistor will be given by:

\[ P = I \times V = 15 \text{ mA} \times 18.8 \text{ V} = 280 \text{ mW} \]

Hence the resistor should be rated at 0.33 W, or greater.

**Diode coding**

The European system for classifying semiconductor diodes involves an alphanumeric code which employs either two letters and three figures (general-purpose diodes) or three letters and two figures (special-purpose diodes). Table 5.7 shows how diodes are coded. Note that the cathode connection of most wire-ended diodes is marked with a stripe.

**Example 5.3**

Identify each of the following diodes:

(a) AA113  
(b) BB105  
(c) BZY88C4V7.

**Solution**

Diode (a) is a general-purpose germanium diode. Diode (b) is a silicon variable capacitance diode. Diode (c) is a silicon zener diode having ±5% tolerance and 4.7 V zener voltage.

**Bipolar junction transistors**

Transistor is short for transfer resistor, a term which provides something of a clue as to how the device operates; the current flowing in the output circuit is determined by the current flowing in the input circuit. Since transistors are three-terminal devices, one electrode must remain common to both the input and the output.
Transistors fall into two main categories: bipolar junction transistors (BJTs) and field effect transistors (FETs), and are also classified according to the semiconductor material employed (silicon or germanium) and to their field of application (e.g. general-purpose, switching, high-frequency, etc.). Various classes of transistor are available according to the application concerned (see Table 5.8).

**BJT operation**

Bipolar junction transistors generally comprise NPN or PNP junctions of either silicon (Si) or germanium (Ge) material (see Figs 5.17 and 5.18). The junctions are, in fact, produced in a single slice of silicon by diffusing impurities through a photographically reduced mask. Silicon transistors are superior when compared with germanium transistors in the vast majority of applications (particularly at high temperatures) and thus germanium devices are very rarely encountered.

Figures 5.19(a) and 5.19(b), respectively, show a simplified representation of NPN and PNP transistors together with their circuit symbols. In either case the electrodes are labelled collector, base and emitter. Note that each junction within the transistor, whether it be collector–base or base–emitter, constitutes a P–N junction. Figures 5.20(a) and 5.20(b), respectively, show the normal bias voltages applied to NPN and PNP transistors. Note that the base–emitter junction is forward biased and the collector–base junction is reverse biased. The base region is, however, made very narrow so that carriers are swept across it from emitter to collector and only a relatively small current flows in the base. To put this into context, the current flowing in the emitter circuit is typically 100 times greater than that flowing in the base. The direction of conventional current flow is from emitter to collector in the case of a PNP

<table>
<thead>
<tr>
<th>Classification</th>
<th>Typical applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-frequency</td>
<td>Transistors designed specifically for audio and low-frequency linear applications (below 100 kHz)</td>
</tr>
<tr>
<td>High-frequency</td>
<td>Transistors designed specifically for radio and wideband linear applications (100 kHz and above)</td>
</tr>
<tr>
<td>Power</td>
<td>Transistors that operate at significant power levels (such devices are often sub-divided into audio and radio frequency types)</td>
</tr>
<tr>
<td>Switching</td>
<td>Transistors designed for switching applications (including power switching)</td>
</tr>
<tr>
<td>Low-noise</td>
<td>Transistors that have low-noise characteristics and which are intended primarily for the amplification of low-amplitude signals</td>
</tr>
<tr>
<td>High-voltage</td>
<td>Transistors designed specifically to handle high voltages</td>
</tr>
<tr>
<td>Driver</td>
<td>Transistors that operate at medium power and voltage levels and which are often used to precede a final (power) stage which operates at an appreciable power level</td>
</tr>
</tbody>
</table>
transistor, and collector to emitter in the case of an NPN device. The equation that relates current flow in the collector, base and emitter currents is:

\[ I_E = I_B + I_C \]

where \( I_E \) is the emitter current, \( I_B \) is the base current and \( I_C \) is the collector current (all expressed in the same units).

**Bipolar transistor characteristics**

The characteristics of a transistor are often presented in the form of a set of graphs relating voltage and current present at the transistor's terminals.

A typical input characteristic (\( I_B \) plotted against \( V_{BE} \)) for a small-signal general-purpose NPN transistor operating in common-emitter mode (see Chapter 7) is shown in Fig. 5.21. This characteristic shows that very little base current flows until the base–emitter voltage (\( V_{BE} \)) exceeds 0.6 V. Thereafter, the base current increases rapidly (this characteristic bears a close resemblance to the forward part of the characteristic for a silicon diode, see Fig. 5.6).

Fig. 5.22 shows a typical output characteristic (\( I_C \) plotted against \( V_{CE} \)) for a small-signal general-purpose NPN transistor operating in common-emitter mode (see Chapter 7). This characteristic
Finally, a typical transfer characteristic ($I_c$ plotted against $I_b$) for a small-signal general purpose NPN transistor operating in common-emitter mode (see Chapter 7) is shown in Fig. 5.23. This characteristic shows an almost linear relationship between collector current and base current (i.e. doubling the value of base current produces double the value of collector current, and so on). This characteristic is reasonably independent of the value of collector-emitter voltage ($V_{CE}$) and thus only a single curve is used.

### Current gain

The current gain offered by a transistor is a measure of its effectiveness as an amplifying device. The most commonly quoted parameter is that which relates to common-emitter mode. In this mode, the input current is applied to the base and the output current appears in the collector (the emitter is effectively common to both the input and output circuits).

The common-emitter current gain is given by:

$$h_{FE} = \frac{I_C}{I_B}$$

where $h_{FE}$ is the hybrid parameter which represents large signal (d.c.) forward current gain, $I_C$ is the collector current and $I_B$ is the base current. When small (rather than large) signal operation is considered, the values of $I_C$ and $I_B$ are incremental (i.e. small changes rather than static values). The current gain is then given by:

$$h_{fe} = \frac{\Delta I_C}{\Delta I_B}$$

where $h_{fe}$ is the hybrid parameter which represents small signal (a.c.) forward current gain, $\Delta I_C$ is the change in collector current which results from a corresponding change in base current, $\Delta I_B$.

Values of $h_{FE}$ and $h_{fe}$ can be obtained from the transfer characteristic ($I_c$ plotted against $I_b$) as shown in Figs 5.23 and 5.24. Note that $h_{FE}$ is found from corresponding static values while $h_{fe}$ is found by measuring the slope of the graph. Also note that, if the transfer characteristic is linear, there is little (if any) difference between $h_{FE}$ and $h_{fe}$.

### Figure 5.22

Typical family of output (collector) characteristics for a small-signal NPN BJT operating in common-emitter mode.

### Figure 5.23

Typical transfer characteristic for a small-signal NPN BJT operating in common-emitter mode.

comprises a family of curves, each relating to a different value of base current ($I_b$). It is worth taking a little time to get familiar with this characteristic as we shall be putting it to good use in Chapter 7. In particular, it is important to note the ‘knee’ that occurs at values of $V_{CE}$ of about 2 V. Also, note how the curves become flattened above this value, with the collector current ($I_C$) not changing very greatly for a comparatively large change in collector–emitter voltage ($V_{CE}$).
Example 5.4
A transistor operates with $I_C = 30 \text{ mA}$ and $I_B = 600 \mu\text{A}$. Determine the value of $I_E$ and $h_{FE}$.

**Solution**

The value of $I_E$ can be calculated from $I_C + I_B$, thus:

$$I_E = I_C + I_B = 30 + 0.6 = 30.6 \text{ mA}$$

The value of $h_{FE}$ can be found from $h_{FE} = I_C / I_B$, thus:

$$h_{FE} = I_C / I_B = 30/0.6 = 50$$

Example 5.5
A transistor operates with a collector current of 97 mA and an emitter current of 98 mA. Determine the value of base current and common-emitter current gain.

**Solution**

Since $I_E = I_C + I_B$, the base current will be given by:

$$I_B = I_E - I_C = 98 - 97 = 1 \text{ mA}$$

The common-emitter current gain ($h_{FE}$) will be given by:

$$h_{FE} = I_C / I_B = 97/1 = 97$$

Example 5.6
An NPN transistor is to be used in a regulator circuit in which a collector current of 1.5 A is to be controlled by a base current of 50 mA. What value of $h_{FE}$ will be required?

If the device is to be operated with $V_{CE} = 6 \text{ V}$, which transistor selected from Table 5.10 would be appropriate for this application and why?
Figure 5.26  Extract from the data sheet for a 2N3702 BJT (courtesy of Fairchild Semiconductor)
Table 5.9 Bipolar transistor parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{C_{\text{max}}}$</td>
<td>The maximum value of collector current</td>
</tr>
<tr>
<td>$V_{CEO_{\text{max}}}$</td>
<td>The maximum value of collector–emitter voltage with the base terminal left open-circuit</td>
</tr>
<tr>
<td>$V_{CBO_{\text{max}}}$</td>
<td>The maximum value of collector–base voltage with the base terminal left open-circuit</td>
</tr>
<tr>
<td>$P_{T_{\text{max}}}$</td>
<td>The maximum total power dissipation</td>
</tr>
<tr>
<td>$h_{FE}$</td>
<td>The large-signal (static) common-emitter current gain</td>
</tr>
<tr>
<td>$h_{ie}$</td>
<td>The small-signal input resistance (see Chapter 7)</td>
</tr>
<tr>
<td>$h_{oe}$</td>
<td>The small-signal output conductance (see Chapter 7)</td>
</tr>
<tr>
<td>$h_{re}$</td>
<td>The small-signal reverse current transfer ratio (see Chapter 7)</td>
</tr>
<tr>
<td>$f_{\text{t typ.}}$</td>
<td>The transition frequency (i.e. the frequency at which the small-signal common-emitter current gain has fallen to unity)</td>
</tr>
</tbody>
</table>

Table 5.10 Characteristics of some common types of bipolar transistor

<table>
<thead>
<tr>
<th>Device</th>
<th>Type</th>
<th>$I_{C_{\text{max}}}$</th>
<th>$V_{CEO_{\text{max}}}$</th>
<th>$V_{CBO_{\text{max}}}$</th>
<th>$P_{T_{\text{max}}}$</th>
<th>$h_{ie}$ at $I_{C}$</th>
<th>$f_{\text{t typ.}}$</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC108</td>
<td>NPN</td>
<td>100 mA</td>
<td>20 V</td>
<td>30 V</td>
<td>300 mW</td>
<td>125</td>
<td>2 mA</td>
<td>250 MHz</td>
</tr>
<tr>
<td>BCY70</td>
<td>PNP</td>
<td>200 mA</td>
<td>−40 V</td>
<td>−50 V</td>
<td>360 mW</td>
<td>150</td>
<td>2 mA</td>
<td>200 MHz</td>
</tr>
<tr>
<td>BD131</td>
<td>NPN</td>
<td>3 A</td>
<td>45 V</td>
<td>70 V</td>
<td>15 W</td>
<td>50</td>
<td>250 mA</td>
<td>60 MHz</td>
</tr>
<tr>
<td>BD132</td>
<td>PNP</td>
<td>3 A</td>
<td>−45 V</td>
<td>−45 V</td>
<td>15 W</td>
<td>50</td>
<td>250 mA</td>
<td>60 MHz</td>
</tr>
<tr>
<td>BF180</td>
<td>NPN</td>
<td>20 mA</td>
<td>20 V</td>
<td>20 V</td>
<td>150 mW</td>
<td>100</td>
<td>10 mA</td>
<td>650 MHz</td>
</tr>
<tr>
<td>2N3053</td>
<td>NPN</td>
<td>700 mA</td>
<td>40 V</td>
<td>60 V</td>
<td>800 mW</td>
<td>150</td>
<td>50 mA</td>
<td>100 MHz</td>
</tr>
<tr>
<td>2N3055</td>
<td>NPN</td>
<td>15 A</td>
<td>60 V</td>
<td>100 V</td>
<td>115 W</td>
<td>50</td>
<td>500 mA</td>
<td>1 MHz</td>
</tr>
<tr>
<td>2N3866</td>
<td>NPN</td>
<td>400 mA</td>
<td>30 V</td>
<td>30 V</td>
<td>3 W</td>
<td>105</td>
<td>50 mA</td>
<td>700 MHz</td>
</tr>
<tr>
<td>2N3904</td>
<td>NPN</td>
<td>200 mA</td>
<td>40 V</td>
<td>60 V</td>
<td>310 mW</td>
<td>150</td>
<td>50 mA</td>
<td>300 MHz</td>
</tr>
</tbody>
</table>

Solution

The required current gain can be found from:

$$h_{FE} = \frac{I_C}{I_B} = \frac{1.5 \, \text{A}}{50 \, \text{mA}} = 30$$

The most appropriate device would be the BD131. The only other device capable of operating at a collector current of 1.5 A would be a 2N3055.

The collector power dissipation will be given by:

$$P_C = I_C \times V_{CE} = 1.5 \, \text{A} \times 6 \, \text{V} = 9 \, \text{W}$$

However, the 2N3055 is rated at 115 W maximum total power dissipation and this is more than ten times the power required.

Example 5.7

A transistor is used in a linear amplifier arrangement. The transistor has small and large signal current gains of 200 and 175, respectively, and bias is arranged so that the static value of collector current is 10 mA. Determine the value of base bias current and the change of output (collector) current that would result from a 10 μA change in input (base) current.

Solution

The value of base bias current can be determined from:

$$I_B = \frac{I_C}{h_{FE}} = \frac{10 \, \text{mA}}{200} = 50 \, \mu\text{A}$$

The change of collector current resulting from a 10 μA change in input current will be given by:

$$\Delta I_C = h_{ie} \times \Delta I_B = 175 \times 10 \, \mu\text{A} = 1.75 \, \text{mA}$$
Field effect transistors

Field effect transistors (FETs) comprise a channel of P-type or N-type material surrounded by material of the opposite polarity. The ends of the channel (in which conduction takes place) form electrodes known as the source and drain. The effective width of the channel (in which conduction takes place) is controlled by a charge placed on the third (gate) electrode. The effective resistance between the source and drain is thus determined by the voltage present at the gate.

FETs are available in two basic forms; junction gate and insulated gate. The gate-source junction of a junction gate field effect transistor (JFET) is effectively a reverse-biased P–N junction. The gate connection of an insulated gate field effect transistor (IGFET), on the other hand, is insulated from the channel and charge is capacitively coupled to the channel. To keep things simple, we will consider only JFET devices in this book. Fig. 5.27 shows the basic construction of an N-channel JFET while Fig. 5.28 shows its symbol and simplified model.

JFETs offer a much higher input resistance when compared with bipolar transistors. For example, the input resistance of a bipolar transistor operating in common-emitter mode (see Chapter 7) is usually around 2.5 kΩ whereas a JFET device operating in equivalent common-source mode (see Chapter 7) would typically exhibit an input resistance of 100 MΩ. This feature makes JFET devices ideal for use in applications where a very high input resistance is desirable.

FET characteristics

As with bipolar transistors, the characteristics of a FET are often presented in the form of a set of graphs relating voltage and current present at the transistor’s terminals. A typical mutual characteristic (I_D plotted against V_GS) for a small-signal general-purpose N-channel JFET operating in common-source mode (see Chapter 7) is shown in Fig. 5.29. This characteristic shows that the drain current is progressively reduced as the gate–source voltage is made more negative. At a certain value of V_GS the drain current falls to zero and the device is said to be cut-off.

Fig. 5.30 shows a typical output characteristic (I_D plotted against V_DS) for a small-signal general-purpose N-channel JFET operating in common-source mode (see Chapter 7). This characteristic
5. Semiconductors

comprises a family of curves, each relating to a different value of gate-source voltage \( V_{GS} \). It is worth taking a little time to get familiar with this characteristic as we shall be using it again in Chapter 7 (you might also like to compare this characteristic with the output characteristic for a transistor operating in common-emitter mode (see Fig. 5.22).

Once again, the characteristic curves have a ‘knee’ that occurs at low values of \( V_{GS} \). Also, note how the curves become flattened above this value with the drain current \( I_D \) not changing very greatly for a comparatively large change in drain–source voltage \( V_{DS} \). These characteristics are, in fact, even flatter than those for a bipolar transistor. Because of their flatness, they are often said to represent a constant current characteristic.

**JFET parameters**

The gain offered by a FET is normally expressed in terms of its forward transfer conductance \( g_{fs} \) or \( Y_{fs} \) in common-source mode. In this mode, the input voltage is applied to the gate and the output current appears in the drain (the source is effectively common to both the input and output circuits).

The common-source forward transfer conductance is given by:

\[
g_{fs} = \frac{\Delta I_D}{\Delta V_{GS}}
\]

where \( \Delta I_D \) is the change in drain current resulting from a corresponding change in gate–source voltage \( \Delta V_{GS} \). The units of forward transfer conductance are siemens (S).

Forward transfer conductance \( g_{fs} \) varies with drain current collector current. For most small-signal devices, \( g_{fs} \) is quoted for values of drain current between 1 mA and 10 mA. It’s also worth noting that most FET parameters (particularly forward transfer conductance) are liable to wide variation from one device to the next. It is, therefore, important to design circuits on the basis of the minimum value for \( g_{fs} \) in order to ensure successful operation with a variety of different devices. JFET parameters are shown in Table 5.11.

The characteristics of several common N-channel field effect transistors are shown in Table 5.12. Figure 5.31 shows a test circuit for obtaining the characteristics of an N-channel FET (the arrangement for a P-channel FET is similar but all meters and supplies must be reversed). A typical N-channel JFET data sheet is shown in Fig. 5.32.

**Example 5.8**

A FET operates with a drain current of 50 mA and a gate–source bias of \(-2 \) V. If the device has a \( g_{fs} \) of 0.025 S, determine the change in drain current if the bias voltage increases to \(-2.5 \) V.
Figure 5.32 Extract from the data sheet for a 2N3819 JFET (courtesy of Fairchild Semiconductor)
5. Semiconductors

Solution

The change in gate-source voltage (\( \Delta V_{gs} \)) is \(-0.5 \) V and the resulting change in drain current can be determined from:

\[
\Delta I_D = \Delta V_{gs} \times g_{fs} = 0.025 \, \text{S} \times -0.5 \, \text{V} = -0.0125 \, \text{A}
\]

This \( \Delta I_D = -12.5 \, \text{mA} \)

The new value of drain current will thus be \((50 \, \text{mA} - 12.5 \, \text{mA})\) or \(37.5 \, \text{mA}\).

The mutual characteristic shown in Fig. 5.33 illustrates this change in drain current. Note how the drain current falls in response to the change in gate–source voltage.

Transistor packages

A wide variety of packaging styles are used for transistors. Small-signal transistors tend to have either plastic packages (e.g., TO92) or miniature metal cases (e.g., TO5 or TO18). Medium- and high-power devices may also be supplied in plastic cases but these are normally fitted with integral metal heat-sinking tabs (e.g., TO126, TO218 or TO220) in order to conduct heat away from the junction. Some older power transistors are supplied in metal cases (either TO3 or TO66). Several popular transistor case styles are shown in Fig. 5.34.

Table 5.11  JFET parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_D ) max.</td>
<td>The maximum value of drain current</td>
</tr>
<tr>
<td>( V_{DS} ) max.</td>
<td>The maximum value of drain–source voltage</td>
</tr>
<tr>
<td>( V_{GS} ) max.</td>
<td>The maximum value of gate–source voltage</td>
</tr>
<tr>
<td>( P_D ) max.</td>
<td>The maximum drain power dissipation</td>
</tr>
<tr>
<td>( g_{fs} )</td>
<td>The common-source forward transfer conductance</td>
</tr>
<tr>
<td>( t_r ) typ.</td>
<td>The typical output rise time in response to a perfect rectangular pulse input</td>
</tr>
<tr>
<td>( t_f ) typ.</td>
<td>The typical output fall time in response to a perfect rectangular pulse input</td>
</tr>
<tr>
<td>( R_{D(on)} ) max.</td>
<td>The maximum value of drain–source resistance when the device is in the conducting (on) state</td>
</tr>
</tbody>
</table>

Table 5.12  Characteristics of some common types of JFET

<table>
<thead>
<tr>
<th>Device</th>
<th>Type</th>
<th>( I_D ) max.</th>
<th>( V_{DS} ) max.</th>
<th>( P_D ) max.</th>
<th>( g_{fs} ) min.</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>2N3819</td>
<td>N-channel</td>
<td>10 mA</td>
<td>25 V</td>
<td>200 mW</td>
<td>4 mS</td>
<td>General purpose</td>
</tr>
<tr>
<td>2N5457</td>
<td>N-channel</td>
<td>10 mA</td>
<td>25 V</td>
<td>310 mW</td>
<td>1 mS</td>
<td>General purpose</td>
</tr>
<tr>
<td>BF244A</td>
<td>N-channel</td>
<td>100 mA</td>
<td>30 V</td>
<td>360 mW</td>
<td>3 mS</td>
<td>RF amplifier</td>
</tr>
<tr>
<td>2N3820</td>
<td>P-channel</td>
<td>–15 mA</td>
<td>20 V</td>
<td>200 mW</td>
<td>0.8 mS</td>
<td>General purpose</td>
</tr>
<tr>
<td>2N5461</td>
<td>P-channel</td>
<td>–9 mA</td>
<td>40 V</td>
<td>310 mW</td>
<td>1.5 mS</td>
<td>Audio amplifiers</td>
</tr>
<tr>
<td>2N5459</td>
<td>N-channel</td>
<td>16 mA</td>
<td>15 V</td>
<td>310 mW</td>
<td>2 mS</td>
<td>General purpose</td>
</tr>
<tr>
<td>J310</td>
<td>N-channel</td>
<td>30 mA</td>
<td>25 V</td>
<td>350 mW</td>
<td>8 mS</td>
<td>VHF/UHF amplifier</td>
</tr>
</tbody>
</table>
Semiconductors

Diodes). With the exception of a few specialized applications (such as amplification at high power levels) integrated circuits have largely rendered a great deal of conventional discrete circuitry obsolete.

Integrated circuits (ICs) are complex circuits fabricated on a small slice of silicon. Integrated circuits may contain as few as 10 or more than 100,000 active devices (transistors and diodes). With the exception of a few specialized applications (such as amplification at high power levels) integrated circuits have largely rendered a great deal of conventional discrete circuitry obsolete.

Integrated circuits can be divided into two general classes, linear (analogue) and digital. Typical examples of linear integrated circuits are operational amplifiers (see Chapter 8) whereas typical examples of digital integrated are logic gates (see Chapter 10).

A number of devices bridge the gap between the analogue and digital world. Such devices include

### Transistor coding

The European system for classifying transistors involves an alphanumeric code which employs either two letters and three figures (general-purpose transistors) or three letters and two figures (special-purpose transistors). Table 5.13 shows how transistors are coded.

#### Example 5.9

Identify each of the following transistors:

(a) AF115
(b) BC109
(c) BD135
(d) BFY51.

#### Solution

(a) Transistor (a) is a general-purpose, low-power, high-frequency germanium transistor.

(b) Transistor (b) is a general-purpose, low-power, low-frequency silicon transistor.

(c) Transistor (c) is a general-purpose, high-power, low-frequency silicon transistor.

(d) Transistor (d) is a special-purpose, low-power, high-frequency silicon transistor.

### Integrated circuits

Integrated circuits (ICs) are complex circuits fabricated on a small slice of silicon. Integrated circuits may contain as few as 10 or more than 100,000 active devices (transistors and diodes). With the exception of a few specialized applications (such as amplification at high power levels) integrated circuits have largely rendered a great deal of conventional discrete circuitry obsolete.

Integrated circuits can be divided into two general classes, linear (analogue) and digital. Typical examples of linear integrated circuits are operational amplifiers (see Chapter 8) whereas typical examples of digital integrated are logic gates (see Chapter 10).

A number of devices bridge the gap between the analogue and digital world. Such devices include
analogue-to-digital converters (ADCs), digital-to-analogue converters (DACs) and timers. For example, the ubiquitous 555 timer contains two operational amplifier stages (configured as voltage comparators) together with a digital bistable stage, a buffer amplifier and an open-collector transistor.

**IC packages**

As with transistors, a variety of different packages are used for integrated circuits. The most popular form of encapsulation used for integrated circuits is the *dual-in-line (DIL)* package which may be fabricated from either plastic or ceramic material (with the latter using a glass hermetic sealant). Common DIL packages have 8, 14, 16, 28 and 40 pins on a 0.1 inch matrix.

Flat package (*flatpack*) construction (featuring both glass–metal and glass–ceramic seals and welded construction) are popular for planar mounting on flat circuit boards. No holes are required to accommodate the leads of such devices which are arranged on a 0.05 inch pitch (i.e. half the pitch used with DIL devices). *Single-in-line (SIL)* and *quad-in-line (QIL)* packages are also becoming increasingly popular while TO5, TO72, TO3 and TO220 encapsulations are also found (the latter being commonly used for three-terminal voltage regulators). Fig. 5.36 shows a variety of common integrated circuit packages while Fig. 5.37 shows a modern *LSI* device.

**Practical investigation**

**Objective**

To obtain the common-emitter transfer characteristic and small-signal current gain for three different NPN bipolar junction transistors.

**Components and test equipment**

Breadboard, digital or analogue meters with d.c. current ranges, 9 V d.c. power supply (or battery), three different NPN transistors (e.g. 2N3904, BC548, BFY50, 2N2222), test leads, connecting wire.
Procedure

Connect the circuit shown in Fig. 5.38. Set the base current meter to the 2 mA range and the collector current meter to the 20 mA range.

Set the variable resistor to minimum position and slowly increase base current until it reaches 0.01 mA. Measure and record the collector current. Repeat for base currents up to 0.1 mA in steps of 0.02 m, at each stage measuring and recording the corresponding value of collector current.

Repeat the investigation with at least two further types of transistor.

Measurements and calculations

For each transistor, record your results in a table showing corresponding values of $I_C$ and $I_B$ (see Table 5.14). Plot graphs showing $I_C$ plotted against $I_B$ (see Fig. 5.39). Calculate the value of $h_{FE}$ for each transistor at $I_C = 2$ mA. Compare your calculated results and characteristic graphs with manufacturer’s data.

Figure 5.38  Transistor test circuit

Table 5.14  Results

<table>
<thead>
<tr>
<th>Base current (mA)</th>
<th>Collector current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2N3904</td>
</tr>
<tr>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.39  Graph layout for plotting the results

Conclusion

Comment on the shape of each graph. Is this what you would expect? Is the graph linear? If not, what will this imply about the static and small-signal values of current gain? Which of the transistors had the highest value of current gain and which had the least value of current gain?

Important formulae introduced in this chapter

LED series resistor:

(page 97)

$$R = \frac{V - V_F}{I}$$

Bipolar transistor currents:

(page 100)

$$I_C = I_B + I_C$$

$$I_B = I_E - I_C$$

$$I_C = I_E - I_B$$

Static current gain for a bipolar transistor:

(page 101)

$$h_{FE} = \frac{I_C}{I_B}$$
5. Semiconductors

Symbols introduced in this chapter

Small-signal current gain for a bipolar transistor:
(page 101)
\[ h_{fe} = \frac{\Delta I_C}{\Delta I_B} \]

Forward transfer conductance for a FET:
(page 106)
\[ g_{fs} = \frac{\Delta I_D}{\Delta V_{GS}} \]

Problems

5.1 Fig. 5.41 shows the characteristics of a diode. What type of material is used in this diode? Give a reason for your answer.

5.2 Use the characteristic shown in Fig. 5.41 to determine the resistance of the diode when (a) \( V_f = 0.65 \) V and (b) \( I_f = 4 \) mA.

Figure 5.41 See Questions 5.1 and 5.2
5.3 The following data refer to a signal diode:

<table>
<thead>
<tr>
<th>$V_F$ (V)</th>
<th>$I_F$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>0.3</td>
<td>1.2</td>
</tr>
<tr>
<td>0.4</td>
<td>3.6</td>
</tr>
<tr>
<td>0.5</td>
<td>6.5</td>
</tr>
<tr>
<td>0.6</td>
<td>10.1</td>
</tr>
<tr>
<td>0.7</td>
<td>13.8</td>
</tr>
</tbody>
</table>

Plot the characteristic and use it to determine:
(a) the forward current when $V_F = 350$ mV;
(b) the forward voltage when $I_F = 15$ mA.

5.4 A diode is marked 'BZY88C9V1'. What type of diode is it? What is its rated voltage? State one application for the diode.

5.5 An LED is to be used to indicate the presence of a 5 V d.c. supply. If the LED has a nominal forward voltage of 2 V, and is rated at a current of 12 mA, determine the value of series resistor required.

5.6 Identify each of the following transistors:
(a) AF117    (b) BC184
(c) BD131    (d) BF180.

5.7 A transistor operates with a collector current of 2.5 A and a base current of 125 mA. Determine the value of emitter current and static common-emitter current gain.

5.8 A transistor operates with a collector current of 98 mA and an emitter current of 103 mA. Determine the value of base current and the static value of common-emitter current gain.

5.9 A bipolar transistor is to be used in a driver circuit in which a base current of 12 mA is available. If the load requires a current of 200 mA, determine the minimum value of common-emitter current gain required.

5.10 An NPN transistor is to operate with $V_{CE} = 10$ V, $I_C = 50$ mA, and $I_B = 400$ μA. Which of the devices listed in Table 5.10 is most suitable for use in this application?

5.11 A transistor is used in a linear amplifier arrangement. The transistor has small- and large-signal current gains of 250 and 220, respectively, and bias is arranged so that the static value of collector current is 2 mA. Determine the value of base bias current and the change of output (collector) current that would result from a 5 μA change in input (base) current.

5.12 The transfer characteristic for an NPN transistor is shown in Fig. 5.42. Use this characteristic to determine:
(a) $I_C$ when $I_B = 50$ μA;
(b) $h_{FE}$ when $I_B = 50$ μA;
(c) $h_{fe}$ when $I_C = 75$ mA.

5.13 The output characteristic of an NPN transistor is shown in Fig. 5.43. Use this characteristic to determine:
(a) $I_C$ when $I_B = 100$ μA and $V_{CE} = 4$ V;
(b) $V_{CE}$ when $I_B = 40$ μA and $I_C = 5$ mA;
(c) $I_B$ when $I_C = 7$ mA and $V_{CE} = 6$ V.

5.14 An N-channel FET operates with a drain current of 20 mA and a gate–source bias of −1 V. If the device has a $g_{fs}$ of 8 mS, determine the new drain current if the bias voltage increases to −1.5 V.

5.15 The following results were obtained during an experiment on an N-channel FET:

<table>
<thead>
<tr>
<th>$V_{GS}$ (V)</th>
<th>$I_D$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>11.4</td>
</tr>
<tr>
<td>−2.0</td>
<td>7.6</td>
</tr>
<tr>
<td>−4.0</td>
<td>3.8</td>
</tr>
<tr>
<td>−6.0</td>
<td>0.2</td>
</tr>
<tr>
<td>−8.0</td>
<td>0.0</td>
</tr>
<tr>
<td>−10.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Plot the mutual characteristic for the FET and use it to determine $g_{fs}$ when $I_D = 5$ A.
5.16 In relation to integrated circuit packages, what do each of the following abbreviations stand for?
(a) DIL
(b) SIL
(c) QIL
(d) LSI.

5.17 Use the data sheet for a 1N4148 diode in Appendix 5 to determine:
(a) the absolute maximum value of reverse repetitive voltage
(b) the maximum power dissipation for the device
(c) the typical reverse current when a reverse voltage of 70 V is applied
(d) the typical forward voltage for a forward current of 5 mA.

5.18 Identify each of the semiconductor devices shown in Fig. 5.44.

5.19 Identify each of the semiconductor devices shown in Fig. 5.45.

5.20 Use the data sheet for a 2N3702 transistor shown in Fig. 5.26 to determine:
(a) the type and class of device
(b) the absolute maximum value of collector–base voltage
(c) the absolute maximum value of collector current
(d) the maximum value of collector–emitter saturation voltage
(e) the maximum value of total power dissipation
(f) the package style.

5.21 A 2N3702 transistor operates under the following conditions: $V_{BE} = 0.7 \text{ V}$, $I_g = 0.5 \text{ mA}$, $V_{CE} = 9 \text{ V}$ and $I_C = 50 \text{ mA}$. Determine whether or not this exceeds the maximum total device dissipation.
5.22 Use the data sheet for a 2N3819 transistor shown in Fig. 5.32 to determine:

(a) the type and class of device
(b) the absolute maximum value of drain–gate voltage
(c) the absolute maximum value of drain current
(d) the maximum value of reverse gate current
(e) the minimum value of forward transfer conductance
(f) the maximum value of input capacitance
(g) the maximum value of total device dissipation
(h) the package style.
5.23 The output characteristic of an N-channel JFET is shown in Fig. 5.46. Use this characteristic to determine:

(a) $I_D$ when $V_{GS} = -2\,\text{V}$ and $V_{DS} = 10\,\text{V}$; 
(b) $V_{DS}$ when $V_{GS} = -1\,\text{V}$ and $I_D = 13\,\text{mA}$; 
(c) $V_{GS}$ when $I_D = 18\,\text{mA}$ and $V_{DS} = 11\,\text{V}$.

5.24 Sketch a circuit diagram showing a test circuit for obtaining the characteristics of a PNP BJT. Label your diagram clearly and indicate the polarity of all supplies and test meters.

5.25 Sketch a circuit diagram showing a test circuit for obtaining the characteristics of a P-channel JFET. Label your diagram clearly and indicate the polarity of all supplies and test meters.

5.26 Sketch a circuit showing how a thyristor can be used to control the current through a resistive load. Label your diagram clearly and explain briefly how the circuit is triggered.

Answers to these problems appear on page 417.
Power supplies

Chapter summary

This chapter deals with the unsung hero of most electronic systems, the power supply. Nearly all electronic circuits require a source of well-regulated d.c. at voltages of typically between 5 V and 30 V. In some cases this supply can be derived directly from batteries (e.g. 6 V, 9 V, 12 V) but in many others it is desirable to make use of a standard a.c. mains outlet. This chapter explains how rectifier and smoothing circuits operate and how power supply output voltages can be closely regulated. The chapter concludes with a brief description of some practical power supply circuits.
The block diagram of a d.c. power supply is shown in Fig. 6.1. Since the mains input is at a relatively high voltage, a step-down transformer of appropriate turns ratio is used to convert this to a low voltage. The a.c. output from the transformer secondary is then rectified using conventional silicon rectifier diodes (see Chapter 5) to produce an unsmoothed (sometimes referred to as pulsating d.c.) output. This is then smoothed and filtered before being applied to a circuit which will regulate (or stabilize) the output voltage so that it remains relatively constant in spite of variations in both load current and incoming mains voltage. Fig. 6.2 shows how some of the electronic components that we have already met can be used in the realization of the block diagram in Fig. 6.1. The iron-cored step-down transformer feeds a rectifier arrangement (often based on a bridge circuit). The output of the rectifier is then applied to a high-value reservoir capacitor. This capacitor stores a considerable amount of charge and is being constantly topped-up by the rectifier arrangement. The capacitor also helps to smooth out the voltage pulses produced by the rectifier. Finally, a stabilizing circuit (often based on a series transistor regulator and a zener diode voltage reference) provides a constant output voltage. We shall now examine each stage of this arrangement in turn, building up to some complete power supply circuits at the end of the chapter.

Rectifiers

Semiconductor diodes (see Chapter 5) are commonly used to convert alternating current (a.c.) to direct current (d.c.), in which case they are referred to as rectifiers. The simplest form of rectifier circuit makes use of a single diode and, since it operates on only either positive or negative half-cycles of the supply, it is known as a half-wave rectifier.

Fig. 6.4 shows a simple half-wave rectifier circuit. Mains voltage (220 to 240 V) is applied to the primary of a step-down transformer (T1). The secondary of T1 steps down the 240 V r.m.s. to 12 V r.m.s. (the turns ratio of T1 will thus
be 240/12 or 20:1). Diode D1 will only allow the current to flow in the direction shown (i.e. from cathode to anode). D1 will be forward biased during each positive half-cycle (relative to common) and will effectively behave like a closed switch. When the circuit current tries to flow in the opposite direction, the voltage bias across the diode will be reversed, causing the diode to act like an open switch (see Figs 6.5(a) and 6.5(b), respectively).

The switching action of D1 results in a pulsating output voltage which is developed across the load resistor ($R_L$). Since the mains supply is at 50 Hz, the pulses of voltage developed across $R_L$ will also be at 50 Hz even if only half the a.c. cycle is present. During the positive half-cycle, the diode will drop the 0.6 V to 0.7 V forward threshold voltage normally associated with silicon diodes. However, during the negative half-cycle the peak a.c. voltage will be dropped across D1 when it is reverse biased. This is an important consideration when selecting a diode for a particular application. Assuming that the secondary of T1 provides 12 V r.m.s., the peak voltage output from the transformer’s secondary winding will be given by:

$$V_{pk} = 1.414 \times V_{r.m.s.} = 1.414 \times 12 \text{ V} = 16.97 \text{ V}$$

The peak voltage applied to D1 will thus be approximately 17 V. The negative half-cycles are blocked by D1 and thus only the positive half-cycles appear across $R_L$. Note, however, that the actual peak voltage across $R_L$ will be the 17 V positive peak being supplied from the secondary on T1, minus the 0.7 V forward threshold voltage dropped by D1. In other words, positive half-cycle pulses having a peak amplitude of 16.3 V will appear across $R_L$.

**Example 6.1**

A mains transformer having a turns ratio of 44:1 is connected to a 220 V r.m.s. mains supply. If the secondary output is applied to a half-wave rectifier, determine the peak voltage that will appear across a load.

**Solution**

The r.m.s. secondary voltage will be given by:

$$V_s = \frac{V_p}{44} = \frac{220}{44} = 5 \text{ V}$$

The peak voltage developed after rectification will be given by:

$$V_{pk} = 1.414 \times 5 \text{ V} = 7.07 \text{ V}$$

Assuming that the diode is a silicon device with a forward voltage drop of 0.6 V, the actual peak voltage dropped across the load will be:

$$V_L = 7.07 \text{ V} - 0.6 \text{ V} = 6.47 \text{ V}$$

**Reservoir and smoothing circuits**

Fig. 6.6 shows a considerable improvement to the circuit of Fig. 6.4. The capacitor, C1, has been added to ensure that the output voltage remains at, or near, the peak voltage even when the diode is not conducting. When the primary voltage is
6 Power supplies

Figure 6.6 A simple half-wave rectifier circuit with reservoir capacitor.

First applied to T1, the first positive half-cycle output from the secondary will charge C1 to the peak value seen across R_L. Hence C1 charges to 16.3 V at the peak of the positive half-cycle. Because C1 and R_L are in parallel, the voltage across R_L will be the same as that across C1.

The time required for C1 to charge to the maximum (peak) level is determined by the charging circuit time constant (the series resistance multiplied by the capacitance value). In this circuit, the series resistance comprises the secondary winding resistance together with the forward resistance of the diode and the (minimal) resistance of the wiring and connections. Hence C1 charges very rapidly as soon as D1 starts to conduct.

The time required for C1 to discharge is, in contrast, very much greater. The discharge time constant is determined by the capacitance value and the load resistance, R_L. In practice, R_L is very much larger than the resistance of the secondary circuit and hence C1 takes an appreciable time to discharge. During this time, D1 will be reverse biased and will thus be held in its non-conducting state. As a consequence, the only discharge path for C1 is through R_L.

C1 is referred to as a reservoir capacitor. It stores charge during the positive half-cycles of secondary voltage and releases it during the negative half-cycles. The circuit of Fig. 6.6 is thus able to maintain a reasonably constant output voltage across R_L. Even so, C1 will discharge by a small amount during the negative half-cycle periods from the transformer secondary.

Figure 6.7 shows the secondary voltage waveform together with the voltage developed across R_L with and without C1 present. This gives rise to a small variation in the d.c. output voltage (known as ripple). Since ripple is undesirable we must take additional precautions to reduce it. One obvious method of reducing the amplitude of the ripple is that of simply increasing the discharge time constant. This can be achieved either by increasing the value of C1 or by increasing the resistance value of R_L. In practice, however, the latter is not really an option because R_L is the effective resistance of the circuit being supplied and we don’t usually have the ability to change it! Increasing the value of C1 is a more practical alternative and very large capacitor values (often in excess of 4,700 μF) are typical.

Fig. 6.8 shows a further refinement of the simple power supply circuit. This circuit employs two additional components, R1 and C1, which act as a filter to remove the ripple. The value of C1 is chosen so that the component exhibits a negligible reactance at the ripple frequency (50 Hz for a half-wave rectifier or 100 Hz for a full-wave.

Figure 6.8 Half-wave rectifier circuit with R–C smoothing filter.
Example 6.2
The $R$–$C$ smoothing filter in a 50 Hz mains operated half-wave rectifier circuit consists of $R_1 = 100 \, \Omega$ and $C_2 = 1,000 \, \mu F$. If 1 V of ripple appears at the input of the circuit, determine the amount of ripple appearing at the output.

Solution
First we must determine the reactance of the capacitor, $C_1$, at the ripple frequency (50 Hz):

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 50 \times 1,000 \times 10^{-6}} = \frac{1,000}{314} = 3.18 \, \Omega$$

The amount of ripple at the output of the circuit (i.e. appearing across $C_1$) will be given by:

$$V_{\text{ripple}} = 1\times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 1\times \frac{3.18}{\sqrt{100^2 + 3.18^2}}$$

From which:

$$V = 0.032 \, V = 32 \, \text{mV}$$

Improved ripple filters
A further improvement can be achieved by using an inductor, $L_1$, instead of a resistor in the smoothing circuit. This circuit also offers the advantage that the minimum d.c. voltage is dropped across the inductor (in the circuit of Fig. 6.7, the d.c. output voltage is reduced by an amount equal to the voltage drop across $R_1$).

Fig. 6.9 shows the circuit of a half-wave power supply with an $L$–$C$ smoothing circuit. At the ripple frequency, $L_1$ exhibits a high value of inductive reactance while $C_1$ exhibits a low value of capacitive reactance. The combined effect is that of an attenuator which greatly reduces the amplitude of the ripple while having a negligible effect on the direct voltage.

Figure 6.9 Half-wave rectifier circuit with $L$–$C$ smoothing filter

Example 6.3
The $L$–$C$ smoothing filter in a 50 Hz mains operated half-wave rectifier circuit consists of $L_1 = 10 \, H$ and $C_2 = 1,000 \, \mu F$. If 1 V of ripple appears at the input of the circuit, determine the amount of ripple appearing at the output.

Solution
Once again, the reactance of the capacitor, $C_1$, is 3.18 $\Omega$ (see Example 6.2). The reactance of $L_1$ at 50 Hz can be calculated from:

$$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 10 = 3,140 \, \Omega$$

The amount of ripple at the output of the circuit (i.e. appearing across $C_1$) will be approximately given by:

$$V = 1\times \frac{X_C}{X_C + X_L} = 1\times \frac{3.18}{3140 + 3.18} = 0.001 \, V$$

Hence the ripple produced by this arrangement (with 1 V of 50 Hz a.c. superimposed on the rectified input) will be a mere 1 mV. It is worth comparing this value with that obtained from the previous example!

Finally, it is important to note that the amount of ripple present at the output of a power supply will increase when the supply is loaded.

Full-wave rectifiers
Unfortunately, the half-wave rectifier circuit is relatively inefficient as conduction takes place only on alternate half-cycles. A better rectifier arrangement would make use of both positive and negative half-cycles. These full-wave rectifier circuits offer a considerable improvement over their half-wave counterparts. They are not only more efficient but are significantly less demanding in terms of the reservoir and smoothing.
components. There are two basic forms of full-wave rectifier: the bi-phase type and the bridge rectifier type.

**Bi-phase rectifier circuits**

Fig. 6.10 shows a simple bi-phase rectifier circuit. Mains voltage (240 V) is applied to the primary of the step-down transformer (T1) which has two identical secondary windings, each providing 12 V r.m.s. (the turns ratio of T1 will thus be 240/12 or 20:1 for each secondary winding).

On positive half-cycles, point A will be positive with respect to point B. Similarly, point B will be positive with respect to point C. In this condition D1 will allow conduction (its anode will be positive with respect to its cathode) while D2 will not allow conduction (its anode will be negative with respect to its cathode). Thus D1 alone conducts on positive half-cycles.

On negative half-cycles, point C will be positive with respect to point B. Similarly, point B will be positive with respect to point A. In this condition D2 will allow conduction (its anode will be positive with respect to its cathode) while D1 will not allow conduction (its anode will be negative with respect to its cathode). Thus D2 alone conducts on negative half-cycles.

Fig. 6.11 shows the bi-phase rectifier circuit with the diodes replaced by switches. In Fig. 6.11(a) D1 is shown conducting on a positive half-cycle while in Fig. 6.11(b) D2 is shown conducting. The result is that current is routed through the load in the same direction on successive half-cycles.

Furthermore, this current is derived alternately from the two secondary windings.

As with the half-wave rectifier, the switching action of the two diodes results in a pulsating output voltage being developed across the load resistor \( R_L \). However, unlike the half-wave circuit the pulses of voltage developed across \( R_L \) will occur at a frequency of 100 Hz (not 50 Hz). This doubling of the ripple frequency allows us to use smaller values of reservoir and smoothing capacitor to obtain the same degree of ripple reduction (recall that the reactance of a capacitor is reduced as frequency increases).

As before, the peak voltage produced by each of the secondary windings will be approximately 17 V and the peak voltage across \( R_L \) will be 16.3 V
Bridge rectifier circuits

An alternative to the use of the bi-phase circuit is that of using a four-diode bridge rectifier (see Fig. 6.14) in which opposite pairs of diodes conduct on alternate half-cycles. This arrangement avoids the need to have two separate secondary windings.

A full-wave bridge rectifier arrangement is shown in Fig. 6.15. Mains voltage (240 V) is applied to the primary of a step-down transformer (T1). The secondary winding provides 12 V r.m.s. (approximately 17 V peak) and has a turns ratio of 20:1, as before. On positive half-cycles, point A will be positive with respect to point B. In this condition D1 and D2 will allow conduction while D3 and D4 will not allow conduction. Conversely, on negative half-cycles, point B will be positive with respect to point A. In this condition D3 and D4 will allow conduction while D1 and D2 will not allow conduction.

Fig. 6.16 shows the bridge rectifier circuit with the diodes replaced by four switches. In Fig. 6.16(a) D1 and D2 are conducting on a positive half-cycle while in Fig. 6.16(b) D3 and
6 Power supplies

Figure 6.15 Full-wave bridge rectifier circuit

Figure 6.16 (a) Bridge rectifier with D1 and D2 conducting, D3 and D4 non-conducting (b) bridge rectifier with D1 and D2 non-conducting, D3 and D4 conducting

D4 are conducting. Once again, the result is that current is routed through the load in the same direction on successive half-cycles. As with the bi-phase rectifier, the switching action of the two diodes results in a pulsating output voltage being developed across the load resistor \( R_L \). Once again, the peak output voltage is approximately 16.3 V (i.e. 17 V less the 0.7 V forward threshold voltage).

Fig. 6.17 shows how a reservoir capacitor (\( C_1 \)) can be added to maintain the output voltage when the diodes are not conducting. This component operates in exactly the same way as for the bi-phase circuit, i.e. it charges to approximately 16.3 V at the peak of the positive half-cycle and holds the voltage at this level when the diodes are in their non-conducting states. This component operates in exactly the same way as for the bi-phase circuit and the secondary and rectified output waveforms are shown in Fig. 6.18. Once again note that the ripple frequency is twice that of the incoming a.c. supply.

Finally, \( R-C \) and \( L-C \) ripple filters can be added to bi-phase and bridge rectifier circuits in exactly the same way as those shown for the half-wave rectifier arrangement (see Figs 6.8 and 6.9).

Voltage regulators

A simple voltage regulator is shown in Fig. 6.19. \( R_s \) is included to limit the zener current to a safe value when the load is disconnected. When a load \( (R_L) \) is connected, the zener current \( (I_Z) \) will fall as current is diverted into the load resistance (it is usual to allow a minimum current of 2 mA to 5 mA in order to ensure that the diode regulates). The output voltage \( (V_Z) \) will remain at the zener voltage until regulation fails at the point at which the potential divider formed by \( R_s \) and \( R_L \).
produces a lower output voltage that is less than \( V_Z \). The ratio of \( R_S \) to \( R_L \) is thus important. At the point at which the circuit just begins to fail to regulate:

\[
V_Z = V_{IN} \times \frac{R_L}{R_L + R_S}
\]

where \( V_{IN} \) is the unregulated input voltage. Thus the maximum value for \( R_S \) can be calculated from:

\[
R_S \text{ max.} = R_L \times \left( \frac{V_{IN}}{V_{IN} - 1} \right)
\]

The power dissipated in the zener diode will be given by \( P_Z = I_Z \times V_Z \), hence the minimum value for \( R_S \) can be determined from the off-load condition when:

\[
R_S \text{ min.} = \frac{V_{IN} - V_Z}{I_Z} = \frac{V_{IN} - V_Z}{P_Z \text{ max.}} = \frac{(V_{IN} - V_Z) \times V_Z}{P_Z \text{ max.}}
\]

Thus:

\[
R_S \text{ min.} = \frac{V_{IN} \times V_Z - V_Z^2}{P_Z \text{ max.}}
\]

where \( P_Z \text{ max.} \) is the maximum rated power dissipation for the zener diode.

**Example 6.4**

A 5 V zener diode has a maximum rated power dissipation of 500 mW. If the diode is to be used in a simple regulator circuit to supply a regulated 5 V to a load having a resistance of 400 Ω, determine a suitable value of series resistor for operation in conjunction with a supply of 9 V.

**Solution**

We shall use an arrangement similar to that shown in Fig. 6.19. First we should determine the maximum value for the series resistor, \( R_S \):

\[
R_S \text{ max.} = R_L \times \left( \frac{V_{IN}}{V_{IN} - 1} \right)
\]

thus:

\[
R_S \text{ max.} = 400 \times \left( \frac{9}{5} - 1 \right) = 400 \times (1.8 - 1) = 320 \Omega
\]

Now we need to determine the minimum value for the series resistor, \( R_S \):

\[
R_S \text{ min.} = \frac{V_{IN} \times V_Z - V_Z^2}{P_Z \text{ max.}}
\]

thus:

\[
R_S \text{ min.} = \frac{(9 \times 5) - 5^2}{0.5} = \frac{45 - 25}{0.5} = 40 \Omega
\]

Hence a suitable value for \( R_S \) would be 150 Ω (roughly mid-way between the two extremes).

**Output resistance and voltage regulation**

In a perfect power supply, the output voltage would remain constant regardless of the current taken by the load. In practice, however, the output voltage falls as the load current increases. To account for this fact, we say that the power supply has **internal resistance** (ideally this should be zero). This internal resistance appears at the output of the supply and is defined as the change in output voltage divided by the corresponding change in output current. Hence:

\[
R_I = \frac{\text{change in output voltage}}{\text{change in output current}} = \frac{\Delta V_{out}}{\Delta I_{out}}
\]

where \( \Delta I_{out} \) represents a small change in output (load) current and \( \Delta V_{out} \) represents a corresponding small change in output voltage. The **regulation** of a power supply is given by the relationship:

\[
\text{Regulation} = \frac{\text{change in output voltage}}{\text{change in line (input) voltage}} \times 100\%
\]

Ideally, the value of regulation should be very small. Simple shunt zener diode regulators of the type shown in Fig. 6.19 are capable of producing values of regulation of 5% to 10%. More sophisticated circuits based on discrete components produce values of between 1% and 5% and integrated circuit regulators often provide values of 1% or less.
6. Power supplies

Example 6.5
The following data were obtained during a test carried out on a d.c. power supply:

(i) Load test
Output voltage (no-load) = 12 V
Output voltage (2 A load current) = 11.5 V

(ii) Regulation test
Output voltage (mains input, 220 V) = 12 V
Output voltage (mains input, 200 V) = 11.9 V

Determine (a) the equivalent output resistance of the power supply and (b) the regulation of the power supply.

Solution
The output resistance can be determined from the load test data:

\[
R_{\text{out}} = \frac{\text{change in output voltage}}{\text{change in output current}} = \frac{12 - 11.5}{2 - 0} = 0.25 \Omega
\]

The regulation can be determined from the regulation test data:

\[
\text{Regulation} = \frac{\text{change in output voltage}}{\text{change in line (input) voltage}} \times 100\%
\]

thus

\[
\text{Regulation} = \frac{12 - 1.9}{220 - 200} \times 100\% = \frac{0.1}{20} \times 100\% = 0.5\%
\]

Practical power supply circuits
Fig. 6.20 shows a simple power supply circuit capable of delivering an output current of up to 250 mA. The circuit uses a full-wave bridge rectifier arrangement (D1 to D4) and a simple C–R filter. The output voltage is regulated by the shunt-connected 12 V zener diode.

Fig. 6.21 shows an improved power supply in which a transistor is used to provide current gain and minimize the power dissipated in the zener diode (TR1 is sometimes referred to as a series-pass transistor). The zener diode, D5, is rated at 13 V and the output voltage will be approximately 0.7 V less than this (i.e. 13 V minus the base-emitter voltage drop associated with TR1). Hence the output voltage is about 12.3 V. The circuit is capable of delivering an output current of up to 500 mA (note that TR1 should be fitted with a small heatsink to conduct away any heat produced). Fig. 6.22 shows a variable power supply. The base voltage to the series-pass transistor is derived from a potentiometer connected across the zener diode, D5. Hence the base voltage is variable from 0 V to 13 V. The transistor requires a substantial heatsink (note that TR1’s dissipation increases as the output voltage is reduced).

Finally, Fig. 6.23 shows a d.c. power supply based on a fixed-voltage three-terminal integrated...
6. Power supplies

Power supplies can be divided into two principal categories, linear and non-linear types. Linear power supplies make use of conventional circuit voltage regulator. These devices are available in standard voltage and current ratings (e.g. 5 V, 12 V, 15 V at 1 A, 2 A and 5 A) and they provide excellent performance in terms of output resistance, ripple rejection and voltage regulation. In addition, such devices usually incorporate overcurrent protection and can withstand a direct short-circuit placed across their output terminals. This is an essential feature in many practical applications!

Voltage multipliers

By adding a second diode and capacitor, we can increase the output of the simple half-wave rectifier arrangement that we met earlier. A voltage doubler using this technique is shown in Fig. 6.25. In this arrangement C1 will charge to the positive peak secondary voltage while C2 will charge to the negative peak secondary voltage. Since the output is taken from C1 and C2 connected in series the resulting output voltage is twice that produced by one diode alone.

The voltage doubler can be extended to produce higher voltages using the cascade arrangement shown in Fig. 6.26. Here C1 charges to the positive peak secondary voltage, while C2 and C3 charge to twice the positive peak secondary voltage. The result is that the output voltage is the sum of the voltages across C1 and C3 which is three times the voltage that would be produced by a single diode. The ladder arrangement shown in Fig. 6.25 can be easily extended to provide even higher voltages but the efficiency of the circuit becomes increasingly impaired and high-order voltage multipliers of this type are only suitable for providing relatively small currents.

Switched mode power supplies

Power supplies can be divided into two principal categories, linear and non-linear types. Linear power supplies make use of conventional...
analogue control techniques – the regulating device operates through a continuous range of current and voltage according to the input and load conditions prevailing at the time. Non-linear power supplies, on the other hand, use digital techniques where the regulating device is switched rapidly ‘on’ and ‘off’ in order to control the mean current and voltage delivered to the load. These non-linear power supplies are commonly referred to as switched mode power supplies or just SMPS. Note that SMPS can be used to step-up (boost) or step-down (buck) the input voltage.

Compared with their conventional linear counterparts, the advantages of SMPS are:

- ability to cope with a very wide input voltage range
- very high efficiency (typically 80% or more)
- compact size and light weight.

The disadvantages of SMPS are:

- relatively complex circuitry
- appreciable noise generated (resulting from the high switching frequency switching action).

Fortunately, the principle of the basic step-down type switched mode regulator is quite straightforward. Take a look at the circuit diagram shown in Fig. 6.27. With S1 closed, the switching diode, D, will be reverse biased and will thus be in a non-conducting state. Current will flow through the inductor, L, charging the capacitor, C, and delivering current to the load, RL.

Current flowing in the inductor, I_L, produces a magnetic flux in its core. When S1 is subsequently opened, the magnetic flux within the inductor rapidly collapses and an e.m.f. is generated across the terminals of the inductor. The polarity of the induced e.m.f. is such that it opposes the original potential and will cause current to continue to flow in the load (i.e. clockwise around the circuit). In this condition, the switching diode, D, will become forward biased, completing the circuit in order to provide a return path for the current.

Waveforms for the circuit of Fig. 6.27 are shown in Fig. 6.28. The inductor voltage, V_L, is alternately positive and negative. When S1 is closed, the voltage dropped across the inductor (from left to right in the diagram) will be equal to +(V_in – V_out).

When S1 is open, the voltage across the inductor will be equal to –V_out (less the small forward voltage drop of the switching diode, D). The average current through the inductor, I_L, is equal to the load current, I_out. Note that a small amount of ripple voltage appears superimposed on the steady output voltage, V_out. This ripple voltage can be reduced by using a relatively large value for the capacitor, C.

In order to control the voltage delivered to the load, V_out, it is simply necessary to adjust the ratio
of ‘on’ to ‘off’ time of the switch. A larger ratio of ‘on’ to ‘off’ time (i.e. a larger duty cycle or mark to space ratio) produces a greater output voltage, and vice versa.

In a practical switched mode power supply, the switch, S1, is replaced by a semiconductor switching device (i.e. a bipolar switching transistor or a MOSFET device). The switching device must have a low ‘on’ resistance and a high ‘off’ resistance and must be capable of switching from the ‘on’ state to the ‘off’ state in a very short time. Regardless of whether it is a bipolar transistor or a MOSFET, the switching device is controlled by a train of rectangular pulses applied to its base or gate terminal. The output voltage can be controlled by varying the width of the pulses in this train. In a practical switched mode power supply, a closed-loop feedback path is employed in which the output voltage is sensed and fed back to the control input of a pulse generator. The result is pulse width modulation (PWM) of the pulse train to the switching device. This pulse width modulation can be achieved using a handful of discrete components or, more usually, is based on a dedicated switched mode controller chip.

The job of controlling a switched mode power supply is an ideal task for an integrated circuit and Fig. 6.30 shows the internal arrangement of a typical example, the LM78S40. This device contains two operational amplifiers (IC1 and IC2) designed to work as comparators, and a two-stage Darlington transistor switch comprising an emitter-follower driver, Q1, and output switch, Q2. The LM78S40 is supplied in a 16-pin dual-in-line (DIL) package.

The LM78S40 can be configured to provide step-up (boost), step-down (buck) and inverting operation. The frequency of the internal current controlled oscillator is set by the value of the capacitor connected to pin 12. Oscillator frequencies of between 100 Hz and 100 kHz are possible but most practical applications operate at frequencies between 20 kHz and 50 kHz. The oscillator duty cycle is internally set to 6:1 but can be varied by means of the current-sensing circuit which normally senses the current in an external resistor connected between pins 12 and 13.

An internal band-gap voltage reference provides a stable voltage reference of 1.3 V at pin 8. The internal reference voltage source is capable of providing a current of up to 10 mA drawn from pin 8. The output transistor, Q2, is capable of carrying a peak current of up to 1.5 A and has
Power supplies

a maximum collector-emitter voltage rating of 40 V. The internal power switching diode, D1, is accessible between pins 1 and 2 and this has similar ratings to Q2. Both D1 and Q2 have switching times of between 300 and 500 ns. IC1 is used to compare the 1.3 V voltage reference (pin 10 connected to pin 8) with a proportion of the output (derived from a simple two-resistor potential divider).

Practical investigation

Objective

To investigate the operation of simple voltage regulators.

Components and test equipment

Breadboard, digital or analogue meters with d.c. voltage and current ranges, 9 V d.c. power supply (or battery), 3.9 V zener diode (e.g. BZX85 or BZY88), NPN TO5 transistor (e.g. 2N3053 or BFY50), 48°C/W TO5 clip-on heatsink, 220 Ω 0.3 W resistor, 15 Ω 0.3 W resistor, 500 Ω and 1 kΩ wirewound variable resistors, connecting wire, test leads.

Procedure

Connect the simple zener diode shunt regulator shown in Fig. 6.32. Set the variable resistor to produce a load current (I_L) of 10 mA then measure and record the output voltage produced across the load, V_L. Repeat for load currents from 20 mA to 100 mA in 10 mA steps.

Connect the transistor regulator shown in Fig. 6.33. Set the variable resistor to produce a load current (I_L) of 25 mA then measure and record the output voltage produced across the load, V_L. Repeat for load current from 50 mA to 250 mA in 25 mA steps.

Measurements and calculations

For each circuit, record your results in a table showing corresponding values of I_L and V_L (see Tables 6.1 and 6.2).

Table 6.1 Table of results for the simple zener voltage regulator

<table>
<thead>
<tr>
<th>Load current (mA)</th>
<th>Output voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Practical investigation

Figure 6.32 Simple zener diode voltage regulator

Figure 6.33 Transistor voltage regulator
Plot graphs showing $V_L$ plotted against $I_L$ for each circuit (see Figures 6.34 and 6.35). By constructing a tangent to each graph, determine the output resistance of each regulator circuit. For the simple shunt zener diode regulator the output resistance should be calculated at $I_L = 30$ mA while for the transistor voltage regulator the output resistance should be calculated at $I_L = 100$ mA.

**Conclusion**

Comment on the shape of each graph. Is this what you would expect? Compare the performance of each circuit and, in particular, the range of load currents over which effective regulation is achieved. Which of the transistors had the lowest value of output resistance? Can you suggest why this is?

**Important formulae introduced in this chapter**

Maximum value of series resistor for a simple shunt zener diode voltage regulator:

$$R_{s\text{ max.}} = R_L \times \left(\frac{V_{SN}}{V_{IN}} - 1\right)$$
6. Power supplies

Minimum value of series resistor for a simple shunt zener diode voltage regulator:
(page 125)
\[ R_s \text{ min.} = \frac{V_{Z \text{max}} - V_Z}{P_{Z \text{max}}}. \]

Output resistance of a power supply:
(page 125)
\[ R_{out} = \frac{\text{change in output voltage}}{\text{change in output current}} = \frac{\Delta V_{out}}{\Delta I_{out}}. \]

Input (line) regulation of a power supply:
(page 125)
\[ \text{Regulation} = \frac{\text{change in output voltage}}{\text{change in line (input) voltage}} \times 100\%. \]

Symbols introduced in this chapter

![Figure 6.36] Circuit symbols introduced in this chapter

Problems

6.1 A half-wave rectifier is fitted with an \( R-C \) smoothing filter comprising \( R = 200 \Omega \) and \( C = 50 \mu F \). If 2 V of 400 Hz ripple appear at the input of the circuit, determine the amount of ripple appearing at the output.

6.2 The \( L-C \) smoothing filter fitted to a 50 Hz mains operated full-wave rectifier circuit consists of \( L = 4 \) H and \( C = 500 \mu F \). If 4 V of ripple appear at the input of the circuit, determine the amount of ripple appearing at the output.

6.3 If a 9 V zener diode is to be used in a simple shunt regulator circuit to supply a load having a nominal resistance of 300 \( \Omega \), determine the maximum value of series resistor for operation in conjunction with a supply of 15 V.

The circuit of a d.c. power supply is shown in Fig. 6.37. Determine the voltages that will appear at test points A, B and C.

In Fig. 6.37, determine the current flowing in \( R_1 \) and the power dissipated in D5 when the circuit is operated without any load connected.

In Fig. 6.37, determine the effect of each of the following fault conditions:
(a) \( R_1 \) open-circuit;
(b) D5 open-circuit;
(c) D5 short-circuit.

A 220 V a.c supply feeds a 20:1 step-down transformer, the secondary of which is connected to a bridge rectifier and reservoir capacitor. Determine the approximate d.c. voltage that will appear.

![Figure 6.37] See Questions 6.4, 6.5 and 6.6
across the reservoir capacitor under 'no-load' conditions.

6.8 The following data were obtained during a load test carried out on a d.c. power supply:
Output voltage (no-load) = 8.5 V
Output voltage (800 mA load) = 8.1 V
Determine the output resistance of the power supply and estimate the output voltage at a load current of 400 mA.

6.9 The following data were obtained during a regulation test on a d.c. power supply:
Output voltage (a.c. input: 230 V) = 15 V
Output voltage (a.c. input: 190 V) = 14.6 V
Determine the regulation of the power supply and estimate the output voltage when the input voltage is 245 V.

6.10 Fig. 6.38 shows a switching regulator circuit that produces an output of 9 V for an input of 4.5 V. What type of regulator is this? Between which pins of IC1 is the switching transistor connected? Which pin on IC1 is used to feed back a proportion of the output voltage to the internal comparator stage?

Answers to these problems appear on page 417.
Amplifiers

Chapter summary

This chapter introduces the basic concepts of amplifiers and amplification. It describes the most common types of amplifier and outlines the basic classes of operation used in both linear and non-linear amplifiers. The chapter also describes methods for predicting the performance of an amplifier based on equivalent circuits and on the use of semiconductor characteristics and load lines. Once again, we conclude with a selection of practical circuits that can be built and tested.
Types of amplifier

Many different types of amplifier are found in electronic circuits. Before we explain the operation of transistor amplifiers in detail, we shall briefly describe the main types of amplifier.

**a.c. coupled amplifiers**

In a.c. coupled amplifiers, stages are coupled together in such a way that d.c. levels are isolated and only the a.c. components of a signal are transferred from stage to stage.

**d.c. coupled amplifiers**

In d.c. (or direct) coupled amplifiers, stages are coupled together in such a way that stages are not isolated to d.c. potentials. Both a.c. and d.c. signal components are transferred from stage to stage.

**Large-signal amplifiers**

Large-signal amplifiers are designed to cater for appreciable voltage and/or current levels (typically from 1 V to 100 V or more).

**Small-signal amplifiers**

Small-signal amplifiers are designed to cater for low-level signals (normally less than 1 V and often much smaller). Small-signal amplifiers have to be specially designed to combat the effects of noise.

**Audio frequency amplifiers**

Audio frequency amplifiers operate in the band of frequencies that is normally associated with audio signals (e.g. 20 Hz to 20 kHz).

**Wideband amplifiers**

Wideband amplifiers are capable of amplifying a very wide range of frequencies, typically from a few tens of hertz to several megahertz.

**Radio frequency amplifiers**

Radio frequency amplifiers operate in the band of frequencies that is normally associated with radio signals (e.g. from 100 kHz to over 1 GHz). Note that it is desirable for amplifiers of this type to be frequency selective and thus their frequency response may be restricted to a relatively narrow band of frequencies (see Fig. 7.9 on page 139).

**Low-noise amplifiers**

Low-noise amplifiers are designed so that they contribute negligible noise (signal disturbance) to the signal being amplified. These amplifiers are usually designed for use with very small signal levels (usually less than 10 mV or so).

**Gain**

One of the most important parameters of an amplifier is the amount of amplification or gain that it provides. Gain is simply the ratio of output voltage to input voltage, output current to input current, or output power to input power (see Fig. 7.2). These three ratios give, respectively, the voltage gain, current gain and power gain. Thus:

\[
A_v = \frac{V_{out}}{V_{in}}
\]

\[
A_i = \frac{I_{out}}{I_{in}}
\]

\[
A_p = \frac{P_{out}}{P_{in}}
\]
Note that, since power is the product of current and voltage \( P = I \times V \), we can infer that:

\[
A_p = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{I_{\text{out}} \times V_{\text{out}}}{I_{\text{in}} \times V_{\text{in}}} = \frac{I_{\text{out}}}{I_{\text{in}}} \times \frac{V_{\text{out}}}{V_{\text{in}}} = A_I \times A_V
\]

**Example 7.1**

An amplifier produces an output voltage of 2 V for an input of 50 mV. If the input and output currents in this condition are, respectively, 4 mA and 200 mA, determine:

(a) the voltage gain;

(b) the current gain;

(c) the power gain.

**Solution**

(a) The voltage gain is calculated from:

\[
A_V = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{2 \text{ V}}{50 \text{ mV}} = 40
\]

(b) The current gain is calculated from:

\[
A_I = \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{200 \text{ mA}}{4 \text{ mA}} = 50
\]

(c) The power gain is calculated from:

\[
A_p = \frac{I_{\text{out}} \times V_{\text{out}}}{I_{\text{in}} \times V_{\text{in}}} = \frac{200 \text{ mA} \times 2 \text{ V}}{4 \text{ mA} \times 50 \text{ mV}} = \frac{0.4 \text{ W}}{200 \mu\text{W}} = 2000
\]

Note that the same result is obtained from:

\[
A_p = A_I \times A_V = 50 \times 40 = 2000
\]

**Class of operation**

An important requirement of most amplifiers is that the output signal should be a faithful copy of the input signal, albeit somewhat larger in amplitude. Other types of amplifier are non-linear, in which case their input and output waveforms will not necessarily be similar. In practice, the degree of linearity provided by an amplifier can be affected by a number of factors including the amount of bias applied (see later) and the amplitude of the input signal.

It is also worth noting that a linear amplifier will become non-linear when the applied input signal exceeds a threshold value. Beyond this value the amplifier is said to be overdriven and the output will become increasingly distorted if the input signal is further increased.

Amplifiers are usually designed to be operated with a particular value of bias supplied to the active devices (i.e. transistors). For linear operation, the active device(s) must be operated in the linear part of their transfer characteristic (\( V_{\text{out}} \) plotted against \( V_{\text{in}} \)). In Fig. 7.3 the input and output signals for an amplifier are operating in linear mode. This form of operation is known as **Class A** and the bias point is adjusted to the mid-point of the linear part of the transfer characteristic. Furthermore, current will flow in the active devices used in a Class A amplifier during a complete cycle of the signal waveform. At no time does the current fall to zero.

Fig. 7.4 shows the effect of moving the bias point down the transfer characteristic and, at the same time, increasing the amplitude of the input signal. From this, you should notice that the extreme negative portion of the output signal has become distorted. This effect arises from the non-linearity of the transfer characteristic that occurs near the origin (i.e. the zero point). Despite the obvious
The output signal will only comprise a series of positive-going half-cycles and the active device(s) will only be conducting during half-cycles of the waveform (i.e. they will only be operating 50% of the time).

This mode of operation is known as **Class B** and is commonly used in high-efficiency push–pull power amplifiers where the two active devices in the output stage operate on alternate half-cycles of the waveform.

Finally, there is one more class of operation to consider. The input and output waveforms for **Class C** operation are shown in Fig. 7.7. Here, the bias point is set at beyond the cut-off (zero) point and a very large input signal is applied. The output waveform will then comprise a series of quite sharp positive-going pulses. These pulses of current or voltage can be applied to a tuned circuit load in order to recreate a sinusoidal signal. In effect, the pulses will excite the tuned circuit non-linearity in the output waveform, the active device(s) will conduct current during a complete cycle of the signal waveform.

Now consider the case of reducing the bias even further while further increasing the amplitude of the input signal (see Fig. 7.5). Here the bias point has been set at the projected cut-off point. The negative-going portion of the output signal becomes cut-off (or clipped) and the active device(s) will cease to conduct for this part of the cycle. This mode of operation is known as **Class AB**.

Now let’s consider what will happen if no bias at all is applied to the amplifier (see Fig. 7.6).

The output signal will only comprise a series of positive-going half-cycles and the active device(s) will only be conducting during half-cycles of the waveform (i.e. they will only be operating 50% of the time).

This mode of operation is known as **Class B** and is commonly used in high-efficiency push–pull power amplifiers where the two active devices in the output stage operate on alternate half-cycles of the waveform.

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7 Amplifiers

Table 7.1 Classes of operation

<table>
<thead>
<tr>
<th>Class of operation</th>
<th>Bias point</th>
<th>Conduction angle (typical)</th>
<th>Efficiency (typical)</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Mid-point</td>
<td>360°</td>
<td>5% to 20%</td>
<td>Linear audio amplifiers</td>
</tr>
<tr>
<td>AB</td>
<td>Projected cut-off</td>
<td>210°</td>
<td>20% to 40%</td>
<td>Push–pull audio amplifiers</td>
</tr>
<tr>
<td>B</td>
<td>At cut-off</td>
<td>180°</td>
<td>40% to 70%</td>
<td>Push–pull audio amplifiers</td>
</tr>
<tr>
<td>C</td>
<td>Beyond cut-off</td>
<td>120°</td>
<td>70% to 90%</td>
<td>Radio frequency power amplifiers</td>
</tr>
</tbody>
</table>

and its inherent flywheel action will produce a sinusoidal output waveform. This mode of operation is only used in RF power amplifiers that must operate at very high levels of efficiency. Table 7.1 summarizes the classes of operation used in amplifiers.

Input and output resistance

Input resistance is the ratio of input voltage to input current and it is expressed in ohms. The input of an amplifier is normally purely resistive (i.e. any reactive component is negligible) in the middle of its working frequency range (i.e. the mid-band). In some cases, the reactance of the input may become appreciable (e.g. if a large value of stray capacitance appears in parallel with the input resistance). In such cases we would refer to input impedance rather than input resistance.

Output resistance is the ratio of open-circuit output voltage to short-circuit output current and is measured in ohms. Note that this resistance is internal to the amplifier and should not be confused with the resistance of a load connected externally.

As with input resistance, the output of an amplifier is normally purely resistive and we can safely ignore any reactive component. If this is not the case, we would once again need to refer to output impedance rather than output resistance.

Fig. 7.8 shows how the input and output resistances are ‘seen’ looking into the input and output terminals, respectively. We shall be returning to this equivalent circuit a little later in this chapter. Finally, it’s important to note that, although these resistances are meaningful in terms of the signals present, they cannot be measured using a conventional meter!

Frequency response

The frequency response characteristics for various types of amplifier are shown in Fig. 7.9. Note that, for response curves of this type, frequency is almost invariably plotted on a logarithmic scale.

The frequency response of an amplifier is usually specified in terms of the upper and lower cut-off frequencies of the amplifier. These frequencies are those at which the output power has dropped to 50% (otherwise known as the −3dB points) or where the voltage gain has dropped to 70.7% of its mid-band value.

Figs 7.10 and 7.11, respectively, show how the bandwidth can be expressed in terms of either power or voltage (the cut-off frequencies, \( f_1 \) and \( f_2 \), and bandwidth are identical).
The mid-band voltage gain corresponds with the flat part of the frequency response characteristic. At that point the voltage gain reaches a maximum of 35 (see Fig. 7.12).

The voltage gain at the two cut-off frequencies can be calculated from:

$$A_v\text{cut-off} = 0.707 \times A_v\text{max} = 0.707 \times 35 = 24.7$$

This value of gain intercepts the frequency response graph at \(f_1=57\) Hz and \(f_2=590\) kHz (see Fig. 7.12).

### Bandwidth

The bandwidth of an amplifier is usually taken as the difference between the upper and lower cut-off frequencies (i.e. \(f_2-f_1\) in Figs 7.10 and 7.11). The bandwidth of an amplifier must be sufficient to accommodate the range of frequencies present within the signals that it is to be presented with. Many signals contain harmonic components (i.e. signals at 2f, 3f, 4f, etc. where f is the frequency of the fundamental signal). To reproduce a square wave, for example, requires an amplifier with a very wide bandwidth (note that a square wave comprises an infinite series of harmonics). Clearly it is not possible to perfectly reproduce such a wave, but it does explain why it can be desirable for an amplifier’s bandwidth to greatly exceed the highest signal frequency that it is required to handle!

### Example 7.2

Determine the mid-band voltage gain and upper and lower cut-off frequencies for the amplifier whose frequency response is shown in Fig. 7.12.
7. Amplifiers

Phase shift
Phase shift is the phase angle between the input and output signal voltages measured in degrees. The measurement is usually carried out in the mid-band where, for most amplifiers, the phase shift remains relatively constant. Note also that conventional single-stage transistor amplifiers provide phase shifts of either 180° or 360°.

Negative feedback
Many practical amplifiers use negative feedback in order to precisely control the gain, reduce distortion and improve bandwidth. The gain can be reduced to a manageable value by feeding back a small proportion of the output. The amount of feedback determines the overall (or closed-loop) gain. Because this form of feedback has the effect of reducing the overall gain of the circuit, this form of feedback is known as negative feedback. An alternative form of feedback, where the output is fed back in such a way as to reinforce the input (rather than to subtract from it) is known as positive feedback. This form of feedback is used in oscillator circuits (see Chapter 9).

Fig. 7.13 shows the block diagram of an amplifier stage with negative feedback applied. In this circuit, the proportion of the output voltage fed back to the input is given by β and the overall voltage gain will be given by:

\[ G = \frac{V_{\text{out}}}{V_{\text{in}}} \]

Figure 7.12 See Example 7.2

Figure 7.13 Amplifier with negative feedback applied
Now \( V_{\text{in}}' = V_{\text{in}} - \beta V_{\text{out}} \) (by applying Kirchhoff’s Voltage Law) (note that the amplifier’s input voltage has been reduced by applying negative feedback) thus:

\[
V_{\text{in}} = V_{\text{in}}' + \beta V_{\text{out}}
\]

and

\[
V_{\text{out}} = A_v \times V_{\text{in}} \quad \text{(note that \( A_v \) is the internal gain of the amplifier)}
\]

Hence:

\[
\text{Overall gain, } G = \frac{A_v \times V_{\text{in}}'}{V_{\text{in}}' + \beta V_{\text{out}} - A_v \times V_{\text{in}}'} = \frac{A_v \times V_{\text{in}}'}{V_{\text{in}}' + \beta (A_v \times V_{\text{in}}')}
\]

Thus:

\[
G = \frac{A_v}{1 + \beta A_v}
\]

Hence, the overall gain with negative feedback applied will be less than the gain without feedback. Furthermore, if \( A_v \) is very large (as is the case with an operational amplifier – see Chapter 8) the overall gain with negative feedback applied will be given by:

\[
G = \frac{1}{\beta} \quad \text{(when } A_v \text{ is very large)}
\]

Note, also, that the loop gain of a feedback amplifier is defined as the product of \( \beta \) and \( A_v \).

**Example 7.3**

An amplifier with negative feedback applied has an open-loop voltage gain of 50, and one-tenth of its output is fed back to the input (i.e. \( \beta = 0.1 \)). Determine the overall voltage gain with negative feedback applied.

**Solution**

With negative feedback applied the overall voltage gain will be given by:

\[
G = \frac{A_v}{1 + \beta A_v} = \frac{50}{1 + (0.1 \times 50)} = \frac{50}{6} = 8.33
\]

**Example 7.4**

If, in Example 7.3, the amplifier’s open-loop voltage gain increases by 20%, determine the percentage increase in overall voltage gain.

**Solution**

The new value of voltage gain will be given by:

\[
A_v' = A_v + 0.2A_v = 1.2 \times 50 = 60
\]
7 Amplifiers

Characteristics, as shown in Tables 7.2 and 7.3 (typical values are given in brackets).

Equivalent circuits

One method of determining the behaviour of an amplifier stage is to make use of an equivalent circuit. Fig. 7.20 shows the basic equivalent circuit of an amplifier. The output circuit is reduced to its Thévenin equivalent (see Chapter 3) comprising a voltage generator ($A_v \times V_{in}$) and a series input resistance, resistance ($R_{out}$). This simple model allows us to forget the complex circuitry that might exist within the amplifier box!
In practice, we use a slightly more complex equivalent circuit model in the analysis of a transistor amplifier. The most frequently used equivalent circuit is that which is based on hybrid parameters (or \( h \)-parameters). In this form of analysis, a transistor is replaced by four components; \( h_i \), \( h_r \), \( h_f \) and \( h_o \) (see Table 7.4). In order to indicate which one of the operating modes is used we add a further subscript letter to each \( h \)-parameter; e for common emitter, b for common base and c for common collector (see Table 7.5). However, to keep things simple, we will only consider common-emitter operation here.

### Common-emitter input resistance (\( h_{ie} \))

The input resistance of a transistor is the resistance that is effectively 'seen' between its input terminals. As such, it is the ratio of the voltage between the input terminals to the current flowing into the input. In the case of a transistor operating in common-emitter mode, the input voltage is the voltage developed between the input terminals.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Common emitter (Fig. 7.14)</th>
<th>Common collector (Fig. 7.15)</th>
<th>Common base (Fig. 7.16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage gain</td>
<td>Medium/high (40)</td>
<td>Unity (1)</td>
<td>High (200)</td>
</tr>
<tr>
<td>Current gain</td>
<td>High (200)</td>
<td>High (200)</td>
<td>Unity (1)</td>
</tr>
<tr>
<td>Power gain</td>
<td>Very high (8000)</td>
<td>High (200)</td>
<td>High (200)</td>
</tr>
<tr>
<td>Input resistance</td>
<td>Medium (2.5 kΩ)</td>
<td>High (100 kΩ)</td>
<td>Low (200 Ω)</td>
</tr>
<tr>
<td>Output resistance</td>
<td>Medium/high (20 kΩ)</td>
<td>Low (100 Ω)</td>
<td>High (110 kΩ)</td>
</tr>
<tr>
<td>Phase shift</td>
<td>180°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Typical applications</td>
<td>General purpose AF and RF amplifiers</td>
<td>Impedance matching; input and output stages</td>
<td>RF and VHF/UHF amplifiers</td>
</tr>
</tbody>
</table>

### Table 7.3 JFET amplifier circuit configurations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Common source (Fig. 7.17)</th>
<th>Common drain (Fig. 7.18)</th>
<th>Common gate (Fig. 7.19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage gain</td>
<td>Medium (40)</td>
<td>Unity (1)</td>
<td>High (250)</td>
</tr>
<tr>
<td>Current gain</td>
<td>Very high (200,000)</td>
<td>Very high (200,000)</td>
<td>Unity (1)</td>
</tr>
<tr>
<td>Power gain</td>
<td>Very high (800,000)</td>
<td>Very high (200,000)</td>
<td>High (250)</td>
</tr>
<tr>
<td>Input resistance</td>
<td>Very high (1 MΩ)</td>
<td>Very high (10 MΩ)</td>
<td>Low (500 Ω)</td>
</tr>
<tr>
<td>Output resistance</td>
<td>Medium/high (50 kΩ)</td>
<td>Low (200 Ω)</td>
<td>High (150 kΩ)</td>
</tr>
<tr>
<td>Phase shift</td>
<td>180°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Typical applications</td>
<td>General purpose AF and RF amplifiers</td>
<td>Impedance matching; input and output stages</td>
<td>RF and VHF/UHF amplifiers</td>
</tr>
</tbody>
</table>

### Table 7.4 General hybrid parameters

\[
h_i = \text{input resistance, } \frac{\Delta V_i}{\Delta I_i}
\]

\[
h_r = \text{reverse voltage transfer ratio, } \frac{\Delta V_{in}}{\Delta V_{out}}
\]

\[
h_f = \text{forward current transfer ratio, } \frac{\Delta I_{out}}{\Delta I_{in}}
\]

\[
h_o = \text{output conductance, } \frac{\Delta I_{out}}{\Delta V_{out}}
\]

### Table 7.5 Common emitter mode h-parameters

\[
h_{ie} = \text{input resistance, } \frac{\Delta V_{be}}{\Delta I_b}
\]

\[
h_{re} = \text{reverse voltage transfer ratio, } \frac{\Delta V_{be}}{\Delta V_{ce}}
\]

\[
h_{fe} = \text{forward current transfer ratio, } \frac{\Delta I_c}{\Delta I_b}
\]

\[
h_{oe} = \text{output conductance, } \frac{\Delta I_c}{\Delta V_{ce}}
\]
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base and emitter, $V_{be}$, while the input current is the current supplied to the base, $I_b$.

Fig. 7.21 shows the current and voltage at the input of a common-emitter amplifier stage while Fig. 7.22 shows how the small-signal input resistance, $R_{in}$, appears between the base and emitter. Note that $R_{in}$ is not a discrete component, it is inside the transistor. From the foregoing we can deduce that:

$$R_{in} = \frac{V_{be}}{I_b}$$

(note that this is similar to the expression for $h_{ie}$).

The transistor’s input characteristic can be used to predict the input resistance of a transistor amplifier stage. Since the input characteristic is non-linear (recall that very little happens until the base-emitter voltage exceeds 0.6 V), the value of input resistance will be very much dependent on the exact point on the graph at which the transistor is being operated. Furthermore, we might expect quite different values of resistance according to whether we are dealing with larger d.c. values or smaller incremental changes (a.c. values). Since this can be a rather difficult concept, it is worth expanding on it.

Fig. 7.23 shows a typical input characteristic in which the transistor is operated with a base current, $I_B$, of 50 $\mu$A. This current produces a base-emitter voltage, $V_{BE}$, of 0.65 V. The input resistance corresponding to these steady (d.c.) values will be given by:

$$R_{in} = \frac{V_{BE}}{I_B} = \frac{0.65 \text{ V}}{50 \text{ } \mu\text{A}} = 13 \text{ k}\Omega$$

Now, suppose that we apply a steady bias current of, say, 70 $\mu$A and superimpose on this a signal that varies above and below this value, swinging through a total change of 100 $\mu$A (i.e. from 20 $\mu$A to 120 $\mu$A). Fig. 7.24 shows that this produces a base-emitter voltage change of 0.05 V. The input resistance seen by this small-signal input current is given by:

$$R_{in} = \frac{\text{change in } V_{be}}{\text{change in } I_B} = \frac{\Delta V_{be}}{\Delta I_B} = \frac{0.05 \text{ V}}{100 \text{ } \mu\text{A}} = 500 \text{ } \Omega$$

In other words:

$$h_{ie} = 500 \text{ } \Omega \quad \text{(since } h_{ie} = \frac{\Delta V_{be}}{\Delta I_B})$$

Figure 7.21 Voltage and current at the input of a common-emitter amplifier stage

Figure 7.22 Input resistance of a common-emitter amplifier stage

Figure 7.23 Using the input characteristic to determine the large-signal (static) input resistance of a transistor connected in common-emitter mode

Figure 7.24 Using the input characteristic to determine the small-signal input resistance of a transistor connected in common-emitter mode
It is worth comparing this value with the steady (d.c.) value. The appreciable difference is entirely attributable to the shape of the input characteristic!

**Common-emitter current gain (h\text{fe})**

The current gain produced by a transistor is the ratio of output current to input current. In the case of a transistor operating in common-emitter mode, the input current is the base current, \( I_b \), while the output current is the collector current, \( I_c \).

Fig. 7.25 shows the small-signal input and output currents and voltages for a common-emitter amplifier stage. The magnitude of the current produced at the output of the transistor is equal to the current gain, \( A_i \), multiplied by the applied base current, \( I_b \). Since the output current is the current flowing in the collector, \( I_c \), we can deduce that:

\[
I_c = A_i \times I_b
\]

where \( A_i = h_{fe} \) (the common-emitter current gain).

Fig. 7.26 shows how this current source appears between the collector and emitter. Once again, the current source is not a discrete component – it appears inside the transistor.

The transistor’s transfer characteristic can be used to predict the current gain of a transistor amplifier stage. Since the transfer characteristic is linear, the current gain remains reasonably constant over a range of collector current. Fig. 7.27 shows a typical transfer characteristic in which the transistor is operated with a base current, \( I_b \), of 240 \( \mu \text{A} \). This current produces a collector current, \( I_c \), of 12 mA. The current gain corresponding to these steady (d.c.) values will be given by:

\[
A_i = \frac{I_c}{I_b} = \frac{2.5 \text{ mA}}{50 \mu \text{A}} = 50
\]

(note that this is similar to the expression for \( h_{fe} \)).

Now, suppose that we apply a steady bias current of, say, 240 \( \mu \text{A} \) and superimpose on this a signal that varies above and below this value, swinging through a total change of 220 \( \mu \text{A} \) (i.e. from 120 \( \mu \text{A} \) to 360 \( \mu \text{A} \)). Fig. 7.28 shows that this produces a collector current swing of 10 mA.

The small-signal a.c. current gain is given by:
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\[ A = \frac{\text{change in } I_c}{\text{change in } I_b} = \frac{\Delta I_c}{\Delta I_b} = \frac{10 \text{ mA}}{220 \mu A} = 45.45 \]

Once again, it is worth comparing this value with the steady state value \((h_{fe})\). Since the transfer characteristic is reasonably linear, the values are quite close (45.45 compared with 50). However, if the transfer characteristic was perfectly linear the value of \(h_{fe}\) would be exactly the same as that for \(h_{FE}\).

\textit{h-parameter equivalent circuit for a transistor in common-emitter mode}

A complete \(h\)-parameter equivalent circuit for a transistor operating in common-emitter mode is shown in Fig. 7.29. We have already shown how the two most important parameters, \(h_{ie}\) and \(h_{fe}\), can be found from the transistor’s characteristic curves. The remaining parameters, \(h_{re}\) and \(h_{oe}\), can, in many applications, be ignored. A typical set of \(h\)-parameters for a BFY50 transistor is shown in Table 7.6. Note how small \(h_{re}\) and \(h_{oe}\) are for a real transistor!

**Example 7.6**

A BFY50 transistor is used in a common-emitter amplifier stage with \(R_L = 10 \text{ k}\Omega\) and \(I_C = 1 \text{ mA}\). Determine the output voltage produced by an input signal of 10 mV. (You may ignore the effect of \(h_{re}\) and any bias components that may be present externally.)

**Solution**

The equivalent circuit (with \(h_{re}\) replaced by a short circuit) is shown in Fig. 7.30. The load effectively appears between the collector and emitter while the input signal appears between the base and emitter. First we need to find the value of input current, \(I_b\), from:

\[ I_b = \frac{V_{in}}{h_{be}} = \frac{10 \text{ mV}}{250 \Omega} = 40 \mu A \]

Next we find the value of current generated, \(I_r\), from:

\[ I_r = h_{re} \times I_b = 80 \times 40 \mu A = 320 \mu A \]

\[ I_c = I_r \times \frac{1}{h_{oe}} \times \frac{1}{h_{re} + R_L} = 320 \mu A \times \frac{1}{80 \times 10^{-6}} \times \frac{1}{(80 \times 10^{-6}) + 10 \text{ k}\Omega} \]

Thus:

\[ I_c = 320 \mu A \times \frac{12.5 \text{ k}\Omega}{12.5 \text{ k}\Omega + 10 \text{ k}\Omega} \]

from which:

\[ I_c = 320 \mu A \times 0.555 = 177.6 \mu A \]

Finally, we can determine the output voltage from:

\[ V_{out} = I_c \times R_L = 177.6 \mu A \times 10 \text{ k}\Omega = 1.776 V \]

\textit{Table 7.6 }\(h\)-parameters for a BFY50 transistor

<table>
<thead>
<tr>
<th>(h_{ie}) ((\Omega))</th>
<th>(h_{re})</th>
<th>(h_{fe})</th>
<th>(h_{oe}) (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.85 \times 10^{-4}</td>
<td>80</td>
<td>35 \times 10^{-6}</td>
</tr>
</tbody>
</table>

Measured at \(I_C = 1 \text{ mA}, V_{CE} = 5 \text{ V}\)

This value of current is shared between the internal resistance between collector and emitter (i.e. \(1/h_{oe}\)) and the external load, \(R_L\). To determine the value of collector current, we can apply the current divider theorem (Chapter 3):

\[ I_c = I_r \times \frac{1}{h_{oe}} \times \frac{1}{h_{re} + R_L} = 320 \mu A \times \frac{1}{80 \times 10^{-6}} \times \frac{1}{(80 \times 10^{-6}) + 10 \text{ k}\Omega} \]

Figure 7.29 \(h\)-parameter equivalent circuit for a transistor amplifier

Figure 7.30 See Example 7.6
**Voltage gain**

We can use hybrid parameters to determine the voltage gain of a transistor stage. We have already shown how the voltage gain of an amplifier stage is given by:

\[ A_V = \frac{V_{out}}{V_{in}} \]

In the case of a common-emitter amplifier stage, \( V_{out} = V_{ce} \) and \( V_{in} = V_{be} \). If we assume that \( h_{ce} \) and \( h_{re} \) are negligible, then:

\[ V_{out} = V_{ce} = I_c \times R_L = I_b \times R_L = h_{ie} \times I_b \times R_L \]

and

\[ V_{in} = V_{be} = I_b \times h_{ie} \]

Thus:

\[ A_V = \frac{V_{out}}{V_{in}} = \frac{h_{ie} \times I_b \times R_L}{I_b \times h_{ie}} = \frac{h_{ie} \times R_L}{h_{ie}} \]

**Example 7.7**

A transistor has \( h_{re} = 150 \) and \( h_{ie} = 1.5 \, \text{k} \Omega \). Assuming that \( h_{re} \) and \( h_{oe} \) are both negligible, determine the value of load resistance required to produce a voltage gain of 200.

**Solution**

Re-arranging \( A_V = \frac{h_{re} \times R_L}{h_{re}} \) to make \( R_L \) the subject gives:

\[ R_L = \frac{A_V \times h_{re}}{h_{re}} \]

For a voltage gain of 200 the value of load resistance can be determined from:

\[ R_L = \frac{200 \times 1.5 \, \text{k} \Omega}{150} = 2 \, \text{k} \Omega \]

**Bias**

We stated earlier that the optimum value of bias for a Class A (linear) amplifier is that value which ensures that the active devices are operated at the midpoint of their transfer characteristics. In practice, this means that a static value of collector current will flow even when there is no signal present. Furthermore, the collector current will flow throughout the complete cycle of an input signal (i.e. conduction will take place over an angle of 360°). At no stage will the transistor be saturated nor should it be cut-off (i.e. the state in which no collector current flows).

In order to ensure that a static value of collector current flows in a transistor, a small current must therefore be applied to the base of the transistor. This current can be derived from the same voltage rail that supplies the collector circuit (via the load). Fig. 7.31 shows a simple Class A common-emitter amplifier circuit in which the base bias resistor, \( R_1 \), and collector load resistor, \( R_2 \), are connected to a common positive supply rail.

The signal is applied to the base terminal of the transistor via a coupling capacitor, \( C_1 \). This capacitor removes the d.c. component of any signal applied to the input terminals and ensures that the base bias current delivered by \( R_1 \) is unaffected by any device connected to the input. \( C_2 \) couples the signal out of the stage and also prevents d.c. current appearing at the output terminals.

In order to stabilize the operating conditions for the stage and compensate for variations in transistor parameters, base bias current for the transistor can be derived from the voltage at the collector (see Fig. 7.32). This voltage is dependent on the collector current which, in turn, depends upon the base current. A negative feedback loop thus exists in which there is a degree of self-regulation. If the collector current increases, the collector voltage will fall and the base current will be reduced. The reduction in base current will produce a corresponding reduction in collector current to offset the original change. Conversely, if the collector current falls, the collector voltage will rise and the base current will increase. This,
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Figure 7.32 An improvement on the circuit shown in Fig. 7.31 (using negative feedback to bias the transistor)

in turn, will produce a corresponding increase in collector current to offset the original change.

The negative feedback path in Fig. 7.32 provides feedback that involves an a.c. (signal) component as well as the d.c. bias. As a result of the a.c. feedback, there is a slight reduction in signal gain. The signal gain can be increased by removing the a.c. signal component from the feedback path so that only the d.c. bias component is present. This can be achieved with the aid of a bypass capacitor, as shown in Fig. 7.33. The value of the bypass capacitor, $C_1$, is chosen so that the component exhibits a very low reactance at the lowest signal frequency when compared with the series base bias resistance, $R_1$. The result of this potential divider arrangement is that the a.c. signal component is effectively bypassed to ground.

Fig. 7.34 shows an improved form of transistor amplifier in which d.c. negative feedback is used to stabilize the stage and compensate for variations in transistor parameters, component values and temperature changes. $R_1$ and $R_2$ form a potential divider that determines the d.c. base potential, $V_b$. The base-emitter voltage ($V_{be}$) is the difference between the potentials present at the base ($V_{bq}$) and emitter ($V_e$). The potential at the emitter is governed by the emitter current ($I_e$). If this current increases, the emitter voltage ($V_e$) will increase and, as a consequence, $V_{be}$ will fall. This, in turn, produces a reduction in emitter current which largely offsets the original change. Conversely, if the emitter current decreases, the emitter voltage ($V_e$) will decrease and $V_{be}$ will increase (remember that $V_b$ remains constant). The increase in bias results in an increase in emitter current compensating for the original change.

**Example 7.8**

Determine the static value of current gain and collector voltage in the circuit shown in Fig. 7.35.

Figure 7.33 Improved version of Fig. 7.31

Figure 7.34 A common-emitter amplifier stage with effective bias stabilization

Figure 7.35 See Example 7.8
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Solution

Since 2 V appears across $R_4$, we can determine the emitter current easily from:

$$I_e = \frac{V_e}{R_4} = \frac{2 \text{ V}}{1 \text{ k}\Omega} = 2 \text{ mA}$$

Next we should determine the base current. This is a little more difficult. The base current is derived from the potential divider formed by $R_1$ and $R_2$. The potential at the junction of $R_1$ and $R_2$ is 2.6 V, hence we can determine the currents through $R_1$ and $R_2$. The difference between these currents will be equal to the base current.

The current in $R_2$ will be given by:

$$I_{R2} = \frac{V_b}{R_2} = \frac{2.6 \text{ V}}{33 \text{ k}\Omega} = 79 \mu\text{A}$$

The current in $R_1$ will be given by:

$$I_{R1} = \frac{9 \text{ V} - V_b}{R_1} = \frac{6.4 \text{ V}}{68 \text{ k}\Omega} = 94.1 \mu\text{A}$$

Hence the base current is found from:

$$I_b = 94.1 \mu\text{A} - 79 \mu\text{A} = 15.1 \mu\text{A}$$

Next we can determine the collector current from:

$$h_{FE} \frac{I_c}{I_b} = \frac{2.0151 \text{ mA}}{15.1 \mu\text{A}} = 133.45$$

Finally we can determine the collector voltage by subtracting the voltage dropped across $R_3$ from the 9 V supply.

The voltage dropped across $R_3$ will be:

$$V_{R3} = I_c \times R_4 = 2.0151 \text{ mA} \times 2.2 \text{ k}\Omega = 4.43 \text{ V}$$

Hence $V_c = 9 \text{ V} - 4.43 \text{ V} = 4.57 \text{ V}$

Predicting Amplifier Performance

The a.c. performance of an amplifier stage can be predicted using a load line superimposed on the relevant set of output characteristics. For a bipolar transistor operating in common-emitter mode the required characteristics are $I_C$ plotted against $V_{CE}$. One end of the load line corresponds to the supply voltage ($V_{CC}$) while the other end corresponds to the value of collector or drain current that would flow with the device totally saturated. In this condition:

$$I_C = \frac{V_{CC}}{R_L}$$

where $R_L$ is the value of collector or drain load resistance.

Fig. 7.36 shows a load line superimposed on a set of output characteristics for a bipolar transistor operating in common-emitter mode. The quiescent point (or operating point) is the point on the load line that corresponds to the conditions that exist when no-signal is applied to the stage. In Fig. 7.36, the base bias current is set at 20 μA so that the quiescent point effectively sits roughly halfway along the load line. This position ensures that the collector voltage can swing both positively (above) and negatively (below) its quiescent value ($V_{CQ}$).

The effect of superimposing an alternating base current (of 20 μA peak–peak) to the d.c. bias current (of 20 μA) can be clearly seen. The corresponding collector current signal can be determined by simply moving up and down the load line.

Example 7.9

The characteristic curves shown in Fig. 7.37 relate to a transistor operating in common-emitter mode. If the transistor is operated with $I_B = 30 \mu\text{A}$, a load resistor of 1.2 kΩ and an 18 V supply, determine the quiescent values of collector voltage and current ($V_{CO}$ and $I_{CO}$). Also determine
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the peak–peak output voltage that would be produced by an input signal of 40 μA peak–peak.

Solution

First we need to construct the load line. The two ends of the load line will correspond to $V_{cc}$ (18 V) on the collector-emitter voltage axis and $(18V/1.2 \text{ kΩ} \text{ or } 15 \text{ mA})$ on the collector current axis. Next we locate the operating point (or quiescent point) from the point of intersection of the $I_B = 30 \mu A$ characteristic and the load line.

Having located the operating point we can read off the quiescent (no-signal) values of collector-emitter voltage ($V_{cq}$) and collector current ($I_{cq}$). Hence:

$V_{cq} = 9.2 \text{ V}$ and $I_{cq} = 7.3 \text{ mA}$

Next we can determine the maximum and minimum values of collector-emitter voltage by locating the appropriate intercept points on Fig. 7.37.

Note that the maximum and minimum values of base current will be $(30 \mu A + 20 \mu A)$ on positive peaks of the signal and $(30 \mu A - 20 \mu A)$ on negative peaks of the signal. The maximum and minimum values of $V_{ce}$ are, respectively, 14.8 V and 3.3 V. Hence the output voltage swing will be $(14.8 \text{ V} - 3.3 \text{ V})$ or 11.5 V peak–peak.

Practical amplifier circuits

The simple common-emitter amplifier stage shown in Fig. 7.38 provides a modest voltage gain (80 to 120 typical) with an input resistance of approximately 1.5 kΩ and an output resistance of around 20 kΩ. The frequency response extends from a few hertz to several hundred kilohertz.

The improved arrangement shown in Fig. 7.39 provides a voltage gain of around 150 to 200 by eliminating the signal frequency negative feedback that occurs through $R1$ in Fig. 7.38.

![Graph showing collector current and collector-emitter voltage](image)

**Figure 7.37** See Example 7.9
Two practical emitter-follower circuits are shown in Figs 7.41 and 7.42. These circuits offer a voltage gain of unity (1) but are ideal for matching a high-resistance source to a low-resistance load. It is important to note that the input resistance varies with the load connected to the output of the circuit (it is typically in the range 50 kΩ to 150 kΩ). The input resistance can be calculated by multiplying $h_{fe}$ by the effective resistance of $R_2$ in parallel with the load connected to the output terminals.

Fig. 7.42 is an improved version of Fig. 7.41 in which the base current is derived from the potential divider formed by $R_1$ and $R_2$. Note, however, that the input resistance is reduced since $R_1$ and $R_2$ effectively appear in parallel with the input. The input resistance of the stage is thus typically in the region of 40 kΩ to 70 kΩ.

Multi-stage amplifiers

In order to provide sufficiently large values of gain, it is frequently necessary to use a number of interconnected stages within an amplifier. The
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The overall gain of an amplifier with several stages (i.e. a multi-stage amplifier) is simply the product of the individual voltage gains. Hence:

\[ A_v = A_{v1} \times A_{v2} \times A_{v3}, \text{ etc.} \]

Note, however, that the bandwidth of a multi-stage amplifier will be less than the bandwidth of each individual stage. In other words, an increase in gain can only be achieved at the expense of a reduction in bandwidth.

Signals can be coupled between the individual stages of a multi-stage amplifier using one of a number of different methods shown in Fig. 7.43.

---

**Power amplifiers**

The term 'power amplifier' can be applied to any amplifier that is designed to deliver an appreciable level of power. There are several important considerations for amplifiers of this type, including the ability to deliver current (as well as voltage) to a load, and also the need to operate with a reasonable degree of efficiency (recall that conventional Class A amplifiers are inefficient).

In order to deliver sufficient current to the load, power amplifiers must have a very low value of output impedance. Thus the final stage (or output stage) is usually based on a device operating in emitter-follower configuration. In order to operate at a reasonable level of efficiency, the output stage must operate in Class AB or
Class B mode (see page 137). One means of satisfying both of these requirements is with the use of a symmetrical output stage based on complementary NPN and PNP devices.

A simple complementary output stage is shown in Fig. 7.45. TR1 is a suitably rated NPN device while TR2 is an identically rated PNP device. Both TR1 and TR2 operate as emitter followers (i.e. common-collector mode) with the output taken from the two emitters, coupled via C2 to the load. In order to bias TR1 and TR2 into Class AB mode two silicon diodes (D1 and D2) are used to provide a constant voltage drop of approximately 1.2 V between the two bases. This voltage drop is required between the bases of TR1 and TR2 in order to bring them to conduction. Since D1 and D2 are both in forward conduction (with current supplied via $R_1$ and $R_2$) they have little effect on the input signal (apart from shifting the d.c. level).

Fig. 7.46 shows an improvement of the basic complementary output stage with the addition of a driver stage (TR1) and a means of adjusting the bias (i.e. operating point) of the two output transistors. VR1 is typically adjusted in order to produce an output stage collector current of between 15 mA and 50 mA (required for Class AB operation). With VR1 set to minimum resistance, the output stage will operate in Class B (this will produce significantly more cross-over distortion because the two devices may both be cut-off for a brief period of each cycle). Fig. 7.47 shows how negative feedback bias (via $R_2$) can be added in order to stabilize the output stage. Practical power amplifiers are shown in Figs 7.48 and 7.49.

**Practical investigation**

**Objective**

To measure the voltage gain and low-frequency response of a simple common-emitter amplifier stage.

**Components and test equipment**

Breadboard, d.c. voltmeter (preferably digital), AF signal generator (with variable frequency sine wave output), two AF voltmeters (or a dual beam oscilloscope), 9 V d.c. power supply (or battery),

![Figure 7.45 A complementary output stage](image)

![Figure 7.46 A complementary output stage with adjustable bias for the output transistors](image)

![Figure 7.47 A complementary output stage with stabilized bias for the driver stage](image)
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Figure 7.48 A simple power amplifier based on a Class AB complementary output stage and Class A driver stage. This amplifier provides an output of 3 W into an 8Ω load and has a frequency response extending from 20 Hz to 50 kHz. Total harmonic distortion (THD) is less than 0.2% at 250 mW output. An input of 2 V peak-peak is required for full output.

Figure 7.49 An improved 3 W audio amplifier. This amplifier has a nominal input resistance of 50 kΩ and a frequency response extending from 30 Hz to 30 kHz at the −3 dB power points. Capacitor C5 is added to ‘roll-off’ the high-frequency response (without this component the high-frequency response extends to around 100 kHz). RV2 is adjusted for a quiescent (no-signal) collector current for TR3 (and TR4) of 25 mA. RV3 is adjusted for a d.c. voltage of exactly 9 V at the junction of the two low-value thermal compensating resistors, R11 and R12. C8 and R13 form a Zobel network to provide frequency compensation of the output load.
Measurements and calculations

Use the measured value of output voltage at 1 kHz in order to determine the voltage gain of the stage.

Graphs

For each set of measurements (470 nF and 10 μF for $C_1$ and $C_2$) plot a graph showing the frequency response of the amplifier stage. In each case, use the graph to determine the low-frequency cut-off.

Conclusion

Comment on the no-signal voltages measured at the collector, base and emitter. Are these what you would expect? Comment on the value of voltage gain that you have obtained. Is this what you would expect? Comment on the shape of each frequency response graph. Explain why there is a difference in cut-off frequency.

Important formulae introduced in this chapter

Voltage gain:

$$A_v = \frac{V_{out}}{V_{in}}$$

(page 135)

Current gain:

$$A_i = \frac{I_{out}}{I_{in}}$$

(page 135)
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Power gain:
(page 135)
\[ A_b = \frac{P_{out}}{P_{in}} \]

Gain with negative feedback applied:
(page 141)
\[ G = \frac{A_v}{1 + \beta A_v} \]

Gain (when \( A_v \) is very large):
(page 141)
\[ G = \frac{1}{\beta} \]

Loop gain:
(page 141)
\[ G_{loop} = \beta A_v \]

Input resistance (common emitter):
(page 143)
\[ h_{ie} = \frac{\Delta V_{ce}}{\Delta I_{ie}} \]

Forward current transfer ratio (common-emitter):
(page 143)
\[ h_{fe} = \frac{\Delta I_c}{\Delta I_b} \]

Output conductance (common-emitter):
(page 143)
\[ h_{oc} = \frac{\Delta I_c}{\Delta V_{ce}} \]

Reverse voltage transfer ratio (common-emitter):
(page 143)
\[ h_{re} = \frac{\Delta V_{ce}}{\Delta V_{eb}} \]

Voltage gain (common-emitter) assuming \( h_{re} \) and \( h_{oc} \) can be neglected:
(page 147)
\[ A_v = \frac{h_{re} \times R_L}{h_{re}} \]

Symbols introduced in this chapter

![Circuit symbols](image)

Figure 7.52 Circuit symbols introduced in this chapter

Problems

7.1 The following measurements were made during a test on an amplifier:
\[ V_{in} = 250 \text{ mV, } I_{in} = 2.5 \text{ mA, } V_{out} = 10 \text{ V, } I_{out} = 400 \text{ mA} \]

Determine:
(a) the voltage gain;
(b) the current gain;
(c) the power gain;
(d) the input resistance.

7.2 An amplifier has a power gain of 25 and identical input and output resistances of 600 \( \Omega \). Determine the input voltage required to produce an output of 10 V.

7.3 Determine the mid-band voltage gain and upper and lower cut-off frequencies for the amplifier whose frequency response curve is shown in Fig. 7.53. Also determine the voltage gain at frequencies of:
(a) 10 Hz
(b) 1 MHz
An amplifier with negative feedback applied has an open-loop voltage gain of 250, and 5% of its output is fed back to the input. Determine the overall voltage gain with negative feedback applied. If the open-loop voltage gain increases by 20% determine the new value of overall voltage gain.

An amplifier produces an open-loop gain of 180. Determine the amount of feedback required if it is to be operated with a precise voltage gain of 50.

A transistor has the following parameters:

- $h_{ie} = 800 \Omega$
- $h_{re}$ is negligible
- $h_{fe} = 120$
- $h_{oe} = 50 \mu S$

If the transistor is to be used as the basis of a common-emitter amplifier stage with $R_L = 12 \, k\Omega$, determine the output voltage when an input signal of 2 mV is applied.

Determine the unknown current and voltages in Fig. 7.54.

The output characteristics of a bipolar transistor are shown in Fig. 7.55. If this transistor is used in an amplifier circuit operating from a 12 V supply with a base bias current of 60 $\mu A$ and a load resistor of 1 $k\Omega$, determine the quiescent values of collector-emitter voltage and collector current. Also determine the peak–peak output voltage produced when an 80 $\mu A$ peak–peak signal current is applied to the base of the transistor.

Fig. 7.56 shows a simple audio power amplifier in which all of the semiconductor devices are silicon and all three transistors have an $h_{FE}$ of 100. If RV1 is adjusted to produce 4.5 V at Test Point D, determine the base, emitter and collector currents and voltages for each transistor and the voltages that will appear at Test Points A to C.

The output characteristics of a junction gate field effect transistor are shown in Fig. 7.57. If this JFET is used in an
7 Amplifiers

7.11 A multi-stage amplifier consists of two R–C coupled common emitter stages. If each stage has a voltage gain of 50, determine the overall voltage gain. Draw a circuit diagram of the amplifier and label your drawing clearly.

7.12 The following r.m.s. voltage measurements were made during a signal test on the simple power amplifier shown in Fig. 7.56 when connected to a 15 \( \Omega \) load:

- \( V_{\text{in}} = 50 \text{ mV} \)
- \( V_{\text{out}} = 2 \text{ V} \)

Determine:

(a) the voltage gain
(b) the output power
(c) the output current.

amplifier circuit operating from an 18 V supply with a gate–source bias voltage of \(-3 \text{ V}\) and a load resistor of 900 \( \Omega \), determine the quiescent values of drain–source voltage and drain current. Also determine the peak–peak output voltage when an input voltage of 2 V peak–peak is applied to the gate. Also determine the voltage gain of the stage.

Figure 7.55 See Question 7.8

Figure 7.56 See Questions 7.9, 7.12 and 7.13
7.13 If the power amplifier shown in Fig. 7.56 produces a maximum r.m.s. output power of 0.25 W, determine its overall efficiency if the supply current is 75 mA. Also determine the power dissipated in each of the output transistors in this condition.

Answers to these problems appear on page 417.
Operational amplifiers

Chapter summary

Operational amplifiers are analogue integrated circuits designed for linear amplification that offer near-ideal characteristics (virtually infinite voltage gain and input resistance coupled with low output resistance and wide bandwidth).

Operational amplifiers can be thought of as universal ‘gain blocks’ to which external components are added in order to define their function within a circuit. By adding two resistors, we can produce an amplifier having a precisely defined gain. Alternatively, with two resistors and two capacitors we can produce a simple band-pass filter. From this you might begin to suspect that operational amplifiers are really easy to use. The good news is that they are!
Symbols and connections

The symbol for an operational amplifier is shown in Fig. 8.2. There are a few things to note about this. The device has two inputs and one output and no common connection. Furthermore, we often don’t show the supply connections – it is often clearer to leave them out of the circuit altogether!

In Fig. 8.2, one of the inputs is marked ‘−’ and the other is marked ‘+’. These polarity markings have nothing to do with the supply connections – they indicate the overall phase shift between each input and the output. The ‘+’ sign indicates zero phase shift while the ‘−’ sign indicates 180° phase shift. Since 180° phase shift produces an inverted waveform, the ‘−’ input is often referred to as the inverting input. Similarly, the ‘+’ input is known as the non-inverting input.

Most (but not all) operational amplifiers require a symmetrical supply (of typically ±6 V to ±15 V) which allows the output voltage to swing both positive (above 0 V) and negative (below 0 V). Fig. 8.3 shows how the supply connections would appear if we decided to include them. Note that we usually have two separate supplies; a positive supply and an equal, but opposite, negative supply. The common connection to these two supplies (i.e. the 0 V supply connection) acts as the common rail in our circuit. The input and output voltages are usually measured relative to this rail.

Operational amplifier parameters

Before we take a look at some of the characteristics of ‘ideal’ and ‘real’ operational amplifiers it is important to define some of the terms and parameters that we apply to these devices.

Open-loop voltage gain

The open-loop voltage gain of an operational amplifier is defined as the ratio of output voltage to input voltage measured with no feedback applied. In practice, this value is exceptionally high (typically greater than 100,000) but is liable to considerable variation from one device to another.

Open-loop voltage gain may thus be thought of as the ‘internal’ voltage gain of the device, thus:

\[ A_{\text{V(OL)}} = \frac{V_{\text{OUT}}}{V_{\text{IN}}} \]

where \( A_{\text{V(OL)}} \) is the open-loop voltage gain, \( V_{\text{OUT}} \) and \( V_{\text{IN}} \) are the output and input voltages, respectively, under open-loop conditions.
8. Operational amplifiers

In linear voltage amplifying applications, a large amount of negative feedback will normally be applied and the open-loop voltage gain can be thought of as the internal voltage gain provided by the device.

The open-loop voltage gain is often expressed in **decibels (dB)** rather than as a ratio. In this case:

\[ A_{\text{VICL}} = 20 \log_{10} \frac{V_{\text{OUT}}}{V_{\text{IN}}} \]

Most operational amplifiers have open-loop voltage gains of 90 dB or more.

**Closed-loop voltage gain**

The closed-loop voltage gain of an operational amplifier is defined as the ratio of output voltage to input voltage measured with a small proportion of the output fed-back to the input (i.e. with feedback applied). The effect of providing negative feedback is to reduce the loop voltage gain to a value that is both predictable and manageable. Practical closed-loop voltage gains range from one to several thousand but note that high values of voltage gain may make unacceptable restrictions on bandwidth (see later).

Closed-loop voltage gain is once again the ratio of output voltage to input voltage but with negative feedback applied, hence:

\[ A_{\text{VCL}} = \frac{V_{\text{OUT}}}{V_{\text{IN}}} \]

where \( A_{\text{VCL}} \) is the open-loop voltage gain, \( V_{\text{OUT}} \) and \( V_{\text{IN}} \) are the output and input voltages, respectively, under closed-loop conditions. The closed-loop voltage gain is normally very much less than the open-loop voltage gain.

**Example 8.1**

An operational amplifier operating with negative feedback produces an output voltage of 2 V when supplied with an input of 400 μV. Determine the value of closed-loop voltage gain.

**Solution**

Now:

\[ A_{\text{VCL}} = \frac{V_{\text{OUT}}}{V_{\text{IN}}} \]

Thus:

\[ A_{\text{VCL}} = \frac{2}{400 \times 10^{-6}} = \frac{2 \times 10^6}{400} = 5,000 \]

Expressed in decibels (rather than as a ratio) this is:

\[ A_{\text{VCL}} = 20 \log_{10} (5,000) = 20 \times 3.7 = 74 \text{ dB} \]

**Input resistance**

The input resistance of an operational amplifier is defined as the ratio of input voltage to input current expressed in ohms. It is often expedient to assume that the input of an operational amplifier is purely resistive, though this is not the case at high frequencies where shunt capacitive reactance may become significant. The input resistance of operational amplifiers is very much dependent on the semiconductor technology employed. In practice values range from about 2 MΩ for common bipolar types to over 10^12 Ω for FET and CMOS devices.

Input resistance is the ratio of input voltage to input current:

\[ R_{\text{IN}} = \frac{V_{\text{IN}}}{I_{\text{IN}}} \]

where \( R_{\text{IN}} \) is the input resistance (in ohms), \( V_{\text{IN}} \) is the input voltage (in volts) and \( I_{\text{IN}} \) is the input current (in amps). Note that we usually assume that the input of an operational amplifier is purely resistive though this may not be the case at high frequencies where shunt capacitive reactance may become significant.

The input resistance of operational amplifiers is very much dependent on the semiconductor technology employed. In practice, values range from about 2 MΩ for bipolar operational amplifiers to over 10^12 Ω for CMOS devices.

**Example 8.2**

An operational amplifier has an input resistance of 2 MΩ. Determine the input current when an input voltage of 5 mV is present.

**Solution**

Now:

\[ R_{\text{IN}} = \frac{V_{\text{IN}}}{I_{\text{IN}}} \]
8. Operational amplifiers

thus

\[
I_{\text{in}} = \frac{V_{\text{in}}}{R_{\text{in}}} = \frac{5 \times 10^{-3}}{2 \times 10^6} = 2.5 \times 10^{-8} \text{A} = 2.5 \text{nA}
\]

**Output resistance**

The output resistance of an operational amplifier is defined as the ratio of open-circuit output voltage to short-circuit output current expressed in ohms. Typical values of output resistance range from less than 10 Ω to around 100 Ω, depending upon the configuration and amount of feedback employed.

Output resistance is the ratio of open-circuit output voltage to short-circuit output current, hence:

\[
R_{\text{out}} = \frac{V_{\text{OUT(OC)}}}{I_{\text{OUT(SC)}}}
\]

where \( R_{\text{out}} \) is the output resistance (in ohms), \( V_{\text{OUT(OC)}} \) is the open-circuit output voltage (in volts) and \( I_{\text{OUT(SC)}} \) is the short-circuit output current (in amps).

**Input offset voltage**

An ideal operational amplifier would provide zero output voltage when 0 V difference is applied to its inputs. In practice, due to imperfect internal balance, there may be some small voltage present at the output. The voltage that must be applied differentially to the operational amplifier input in order to make the output voltage exactly zero is known as the input offset voltage.

Input offset voltage may be minimized by applying relatively large amounts of negative feedback or by using the offset null facility provided by a number of operational amplifier devices. Typical values of input offset voltage range from 1 mV to 15 mV. Where a.c. rather than d.c. coupling is employed, offset voltage is not normally a problem and can be happily ignored.

**Full-power bandwidth**

The full-power bandwidth for an operational amplifier is equivalent to the frequency at which the maximum undistorted peak output voltage swing falls to 0.707 of its low-frequency (d.c.) value (the sinusoidal input voltage remaining constant). Typical full-power bandwidths range from 10 kHz to over 1 MHz for some high-speed devices.

**Slew rate**

Slew rate is the rate of change of output voltage with time, when a rectangular step input voltage is applied (as shown in Fig. 8.4). The slew rate of an operational amplifier is the rate of change of output voltage with time in response to a perfect step-function input. Hence:

\[
\text{Slew rate} = \frac{\Delta V_{\text{OUT}}}{\Delta t}
\]

where \( \Delta V_{\text{OUT}} \) is the change in output voltage (in volts) and \( \Delta t \) is the corresponding interval of time (in seconds).

Slew rate is measured in V/s (or V/μs) and typical values range from 0.2 V/μs to over 20 V/μs. Slew rate imposes a limitation on circuits in which large amplitude pulses rather than small amplitude sinusoidal signals are likely to be encountered.

**Operational amplifier characteristics**

Having defined the parameters that we use to describe operational amplifiers we shall now
consider the desirable characteristics for an ‘ideal’ operational amplifier. These are:

(a) The open-loop voltage gain should be very high (ideally infinite).

(b) The input resistance should be very high (ideally infinite).

(c) The output resistance should be very low (ideally zero).

(d) Full-power bandwidth should be as wide as possible.

(e) Slew rate should be as large as possible.

(f) Input offset should be as small as possible.

The characteristics of most modern integrated circuit operational amplifiers (i.e. ‘real’ operational amplifiers) come very close to those of an ‘ideal’ operational amplifier, as witnessed by the data shown in Table 8.1.

### Table 8.1 Comparison of operational amplifier parameters for ‘ideal’ and ‘real’ devices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ideal</th>
<th>Real</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage gain</td>
<td>Infinite</td>
<td>100,000</td>
</tr>
<tr>
<td>Input resistance</td>
<td>Infinite</td>
<td>100 MΩ</td>
</tr>
<tr>
<td>Output resistance</td>
<td>Zero</td>
<td>20 Ω</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>Infinite</td>
<td>2 MHz</td>
</tr>
<tr>
<td>Slew rate</td>
<td>Infinite</td>
<td>10 V/µs</td>
</tr>
<tr>
<td>Input offset</td>
<td>Zero</td>
<td>Less than 5 mV</td>
</tr>
</tbody>
</table>

**Example 8.3**

A perfect rectangular pulse is applied to the input of an operational amplifier. If it takes 4 µs for the output voltage to change from –5 V to +5 V, determine the slew rate of the device.

**Solution**

The slew rate can be determined from:

\[
\text{Slew rate} = \frac{\Delta V_{\text{OUT}}}{\Delta t} = \frac{10 \text{ V}}{4 \mu\text{s}} = 2.5 \text{ V/µs}
\]

**Example 8.4**

A wideband operational amplifier has a slew rate of 15 V/µs. If the amplifier is used in a circuit with a voltage gain of 20 and a perfect step input of 100 mV is applied to its input, determine the time taken for the output to change level.

**Solution**

The output voltage change will be 20 × 100 = 2,000 mV (or 2 V). Re-arranging the formula for slew rate gives:

\[
\Delta t = \frac{\Delta V_{\text{OUT}}}{\text{Slew rate}} = \frac{2 \text{ V}}{15 \text{ V/µs}} = 0.133 \mu\text{s}
\]

### Operational amplifier applications

Table 8.2 shows abbreviated data for some common types of integrated circuit operational amplifier together with some typical applications.

**Example 8.5**

Which of the operational amplifiers in Table 8.2 would be most suitable for each of the following applications:

(a) amplifying the low-level output from a piezoelectric vibration sensor

(b) a high-gain amplifier that can be used to faithfully amplify very small signals
(c) a low-frequency amplifier for audio signals.

Solution
(a) AD548 (this operational amplifier is designed for use in instrumentation applications and it offers a very low input offset current which is important when the input is derived from a piezoelectric transducer).
(b) CA3140 (this is a low-noise operational amplifier that also offers high gain and fast slew rate).
(c) LM348 or LM741 (both are general-purpose operational amplifiers and are ideal for non-critical applications such as audio amplifiers).

Gain and bandwidth

It is important to note that the product of gain and bandwidth is a constant for any particular operational amplifier. Hence, an increase in gain can only be achieved at the expense of bandwidth, and vice versa.

Fig. 8.5 shows the relationship between voltage gain and bandwidth for a typical operational amplifier (note that the axes use logarithmic rather than linear scales). The open-loop voltage gain (i.e. that obtained with no feedback applied) is 100,000 (or 100 dB) and the bandwidth obtained in this condition is a mere 10 Hz. The effect of applying increasing amounts of negative feedback (and consequently reducing the gain to a more manageable amount) is that the bandwidth increases in direct proportion.

The frequency response curves in Fig. 8.5 show the effect on the bandwidth of making the closed-loop gains equal to 10,000, 1,000, 100, and 10. Table 8.3 summarizes these results. You should also note that the \( V_{\text{OUT}} \) of this amplifier is \( 1 \times 10^6 \) Hz (i.e. 1 MHz).

We can determine the bandwidth of the amplifier when the closed-loop voltage gain is set to 46 dB by constructing a line and noting the intercept point on the response curve. This shows that the bandwidth will be 10 kHz (note that, for this operational amplifier, the \( V_{\text{OUT}} \) product is \( 2 \times 10^6 \) Hz (or 2 MHz).

**Inverting amplifier with feedback**

Fig. 8.6 shows the circuit of an inverting amplifier with negative feedback applied. For the sake of our explanation we will assume that the operational amplifier is ‘ideal’. Now consider what happens when a small positive input voltage is applied. This voltage \( V_{\text{IN}} \) produces a current \( I_{\text{IN}} \) flowing in the input resistor \( R_1 \).

Since the operational amplifier is ‘ideal’ we will assume that:
(a) the input resistance (i.e. the resistance that appears between the inverting and non-inverting input terminals, \( R_{\text{IC}} \)) is infinite
(b) the open-loop voltage gain (i.e. the ratio of \( V_{\text{OUT}} \) to \( V_{\text{IN}} \) with no feedback applied) is infinite.

As a consequence of (a) and (b):
Operational amplifiers

(i) the voltage appearing between the inverting and non-inverting inputs \( V_{IC} \) will be zero, and
(ii) the current flowing into the chip \( I_{IC} \) will be zero (recall that \( I_{IC} = \frac{V_{IC}}{R_{IC}} \) and \( R_{IC} \) is infinite). Applying Kirchhoff’s Current Law at node A gives:
\[
I_{IN} = I_{IC} + I_F \quad \text{but} \quad I_{IC} = 0 \quad \text{thus} \quad I_{IN} = I_F \quad (1)
\]
(this shows that the current in the feedback resistor, \( R_2 \), is the same as the input current, \( I_{IN} \)).

Applying Kirchhoff’s Voltage Law to loop A gives:
\[
V_{IN} = (I_{IN} \times R_1) + V_{IC} \quad \text{but} \quad V_{IC} = 0 \quad \text{thus} \quad V_{IN} = I_{IN} \times R_1 \quad (2)
\]

Using Kirchhoff’s Voltage Law in loop B gives:
\[
V_{OUT} = -V_{IC} + (I_{F} \times R_2) \quad \text{but} \quad V_{IC} = 0 \quad \text{thus} \quad V_{OUT} = I_{F} \times R_2 \quad (3)
\]

Combining (1) and (3) gives:
\[
V_{OUT} = I_{IN} \times R_2 \quad (4)
\]

The voltage gain of the stage is given by:
\[
A_V = \frac{V_{OUT}}{V_{IN}} \quad (5)
\]

Combining (4) and (2) with (5) gives:
\[
A_V = \frac{I_{IN} \times R_2}{I_{IN} \times R_1} = \frac{R_2}{R_1}
\]

To preserve symmetry and minimize offset voltage, a third resistor is often included in series with the non-inverting input. The value of this resistor should be equivalent to the parallel combination of \( R_1 \) and \( R_2 \). Hence:
\[
R_3 = \frac{R_1 \times R_2}{R_1 + R_2}
\]

From this point onwards (and to help you remember the function of the resistors) we shall refer to the input resistance as \( R_{IN} \) and the feedback resistance as \( R_F \) (instead of the more general and less meaningful \( R_1 \) and \( R_2 \), respectively).

Operational amplifier configurations

The three basic configurations for operational voltage amplifiers, together with the expressions for their voltage gain, are shown in Fig. 8.7. Supply rails have been omitted from these diagrams for clarity but are assumed to be symmetrical about 0 V.

All of the amplifier circuits described previously have used direct coupling and thus have frequency response characteristics that extend to d.c. This, of course, is undesirable for many applications, particularly where a wanted a.c. signal may be superimposed on an unwanted d.c. voltage level or when the bandwidth of the amplifier greatly exceeds that of the signal that it is required to amplify. In such cases, capacitors of appropriate value may be inserted in series with the input resistor, \( R_{IN} \), and in parallel with the feedback resistor, \( R_F \), as shown in Fig. 8.8.

The value of the input and feedback capacitors, \( C_{IN} \) and \( C_F \) respectively, are chosen so as to roll-off the frequency response of the amplifier at the desired lower and upper cut-off frequencies, respectively. The effect of these two capacitors on an operational amplifier’s frequency response is shown in Fig. 8.9.

By selecting appropriate values of capacitor, the frequency response of an inverting operational voltage amplifier may be very easily tailored to suit a particular set of requirements.

The lower cut-off frequency is determined by the value of the input capacitance, \( C_{IN} \), and input resistance, \( R_{IN} \). The lower cut-off frequency is given by:
\[
f_l = \frac{1}{2 \pi C_{IN} R_{IN}} = \frac{0.159}{C_{IN} R_{IN}}
\]

where \( f_l \) is the lower cut-off frequency in hertz, \( C_{IN} \) is in farads and \( R_{IN} \) is in ohms.

Provided the upper frequency response it not limited by the gain x bandwidth product, the
8. Operational amplifiers

**Operational amplifiers**

The upper cut-off frequency will be determined by the feedback capacitance, \( C_F \), and feedback resistance, \( R_F \), such that:

\[
f_2 = \frac{1}{2\pi C_F R_F} = 0.159 \frac{1}{C_F R_F}
\]

where \( f_2 \) is the upper cut-off frequency in hertz, \( C_F \) is in farads and \( R_F \) is in ohms.

**Example 8.6**

An inverting operational amplifier is to operate according to the following specification:
- Voltage gain = 100
- Input resistance (at mid-band) = 10 kΩ
- Lower cut-off frequency = 250 Hz
- Upper cut-off frequency = 15 kHz

Devise a circuit to satisfy the above specification using an operational amplifier.

**Solution**

To make things a little easier, we can break the problem down into manageable parts. We shall base our circuit on a single operational amplifier configured as an inverting amplifier with capacitors to define the upper and lower cut-off frequencies, as shown in Fig. 8.9.

The nominal input resistance is the same as the value for \( R_{\text{in}} \). Thus:

\[
R_{\text{in}} = 10 \text{ kΩ}
\]

To determine the value of \( R_F \), we can make use of the formula for mid-band voltage gain:

\[
A_v = \frac{R_2}{R_1}
\]
thus $R_2 = A_v \times R_1 = 100 \times 10 \, \text{k}\Omega = 100 \, \text{k}\Omega$

To determine the value of $C_{\text{in}}$ we will use the formula for the low-frequency cut-off:

$$f_c = \frac{0.159}{C_{\text{in}}R_{\text{in}}}$$

from which:

$$C_{\text{in}} = \frac{0.159}{f_c R_{\text{in}}} = \frac{0.159}{250 \times 10 \times 10^3}$$

hence:

$$C_{\text{in}} = \frac{0.159}{2.5 \times 10^6} = 63 \times 10^{-9} \, \text{F} = 63 \, \text{nF}$$

Finally, to determine the value of $C_F$ we will use the formula for high-frequency cut-off:

$$f_c = \frac{0.159}{C_{\text{F}}R_{\text{in}}}$$

from which:

$$C_F = \frac{0.159}{f_c R_{\text{in}}} = \frac{0.159}{15 \times 10^3 \times 100 \times 10^3}$$

hence:

$$C_F = \frac{0.159}{1.5 \times 10^9} = 106 \times 10^{-9} \, \text{F} = 106 \, \text{pF}$$

For most applications the nearest preferred values (68 nF for $C_{\text{in}}$ and 100 pF for $C_F$) would be perfectly adequate. The complete circuit of the operational amplifier stage is shown in Fig. 8.10.

**Operational amplifier circuits**

As well as their application as a general-purpose amplifying device, operational amplifiers have a number of other uses, including voltage followers, differentiators, integrators, comparators and summing amplifiers. We shall conclude this section by taking a brief look at each of these applications.

**Voltage followers**

A voltage follower using an operational amplifier is shown in Fig. 8.11. This circuit is essentially an inverting amplifier in which 100% of the output is fed back to the input. The result is an amplifier that has a voltage gain of 1 (i.e. unity), a very high input resistance and a very high output resistance. This stage is often referred to as a buffer and is used for matching a high-impedance circuit to a low-impedance circuit.

Typical input and output waveforms for a voltage follower are shown in Fig. 8.12. Notice how the input and output waveforms are both in-phase (they rise and fall together) and that they are identical in amplitude.
8. Operational amplifiers

**Differentiators**

A differentiator using an operational amplifier is shown in Fig. 8.13. A differentiator produces an output voltage that is equivalent to the rate of change of its input. This may sound a little complex but it simply means that if the input voltage remains constant (i.e. if it isn’t changing) the output also remains constant. The faster the input voltage changes the greater will the output be. In mathematics this is equivalent to the differential function.

Typical input and output waveforms for a differentiator are shown in Fig. 8.14. Notice how the square wave input is converted to a train of short duration pulses at the output. Note also that the output waveform is inverted because the signal has been applied to the inverting input of the operational amplifier.

**Integrators**

An integrator using an operational amplifier is shown in Fig. 8.15. This circuit provides the opposite function to that of a differentiator (see earlier) in that its output is equivalent to the area under the graph of the input function rather than its rate of change. If the input voltage remains constant (and is other than 0 V) the output voltage will ramp up or down according to the polarity of the input. The longer the input voltage remains at a particular value the larger the value of output voltage (of either polarity) will be produced.

Typical input and output waveforms for an integrator are shown in Fig. 8.16. Notice how the square wave input is converted to a wave that has a triangular shape. Once again, note that the output waveform is inverted.

**Comparators**

A comparator using an operational amplifier is shown in Fig. 8.17. Since no negative feedback has been applied, this circuit uses the maximum gain of the operational amplifier. The output voltage produced by the operational amplifier will...
thus rise to the maximum possible value (equal to the positive supply rail voltage) whenever the voltage present at the non-inverting input exceeds that present at the inverting input. Conversely, the output voltage produced by the operational amplifier will fall to the minimum possible value (equal to the negative supply rail voltage) whenever the voltage present at the inverting input exceeds that present at the non-inverting input.

Typical input and output waveforms for a comparator are shown in Fig. 8.18. Notice how the output is either +15 V or –15 V depending on the relative polarity of the two inputs. A typical application for a comparator is that of comparing a signal voltage with a reference voltage. The output will go high (or low) in order to signal the result of the comparison.

**Summing amplifiers**

A summing amplifier using an operational amplifier is shown in Fig. 8.19. This circuit produces an output that is the sum of its two input voltages. However, since the operational amplifier is connected in inverting mode, the output voltage is given by:

$$V_{\text{OUT}} = -(V_1 + V_2)$$

where \(V_1\) and \(V_2\) are the input voltages (note that all of the resistors used in the circuit have the same value). Typical input and output waveforms for a summing amplifier are shown in Fig. 8.20. A typical application is that of ‘mixing’ two input signals to produce an output voltage that is the sum of the two.
Positive versus negative feedback

We have already shown how negative feedback can be applied to an operational amplifier in order to produce an exact value of gain. Negative feedback is frequently used in order to stabilize the gain of an amplifier and also to increase the frequency response (recall that, for an amplifier the product of gain and bandwidth is a constant). Positive feedback, on the other hand, results in an increase in gain and a reduction in bandwidth. Furthermore, the usual result of applying positive feedback is that an amplifier becomes unstable and oscillates (i.e. it generates an output without an input being present!). For this reason, positive feedback is only used in amplifiers when the voltage gain is less than unity.

The important thing to remember from all of this is that, when negative feedback is applied to an amplifier the overall gain is reduced and the bandwidth is increased (note that the gain x bandwidth product remains constant). When positive feedback is applied to an amplifier the overall gain increases and the bandwidth is reduced. In most cases this will result in instability and oscillation.

Multi-stage amplifiers

Multi-stage amplifiers can easily be produced using operational amplifiers. Coupling methods can be broadly similar to those described earlier in Chapter 7 (see page 152). As an example, Fig. 8.21 shows a two-stage amplifier in which each stage has a tailored frequency response. Note how $C_1$ and $C_3$ provide d.c. isolation between the stages as well as helping to determine the low-frequency roll-off.

Practical investigation

Objective

To measure the voltage gain and frequency response of an inverting operational amplifier.

Components and test equipment

Breadboard, AF signal generator (with variable-frequency sine wave output), two AF voltmeters (or a dual-beam oscilloscope), ±9 V d.c. power supply (or two 9 V batteries), TL081 (or similar operational amplifier), 22 pF, 2.2 nF, 47 nF and 220 nF capacitors, resistors of $10 \, \text{k}\Omega$ and $100 \, \text{k}\Omega$, 5% 0.25 W, test leads, connecting wire.

Procedure

Connect the circuit shown in Fig. 8.22 with $C_{\text{IN}} = 47 \, \text{nF}$ and $C_{\text{F}} = 2.2 \, \text{nF}$, set the signal generator to produce an output of 100 mV at 1 kHz. Measure and record the output voltage produced and repeat this measurement for frequencies over the range 10 Hz to 100 kHz (see Table 8.4).

Replace $C_{\text{IN}}$ and $C_{\text{F}}$ with 220 nF and 22 pF capacitors and repeat the measurements, this time over the extended frequency range from 1 Hz to 1 MHz, recording your results as shown in Table 8.5.

Measurements and graphs

Use the measured value of output voltage at 1 kHz for both sets of measurements, in order to determine the mid-band voltage gain of the stage. For each set of measurements plot graphs.

Figure 8.21 A multi-stage amplifier (both stages have tailored frequency responses)

Figure 8.22 See Practical investigation
8 Operational amplifiers

showing the frequency response of the amplifier stage (see Fig. 8.23). In each case, use the graph to determine the lower and upper cut-off frequencies.

Calculations

For each circuit calculate:
(a) the mid-band voltage gain
(b) the lower cut-off frequency
(c) the upper cut-off frequency.

Compare the calculated values with the measured values.

Table 8.4 Results ($C_{IN} = 47 \text{ nF}, C_F = 2.2 \text{ nF}$)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Output voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<tr>
<td>20</td>
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<td>200 k</td>
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<td>400 k</td>
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</tbody>
</table>

Conclusion

Comment on the performance of the amplifier stage. Is this what you would expect? Do the measured values agree with those obtained by calculation? If not, suggest reasons for any differences. Suggest typical applications for the circuit.

Table 8.5 Results ($C_{IN} = 220 \text{ nF}, C_F = 22 \text{ pF}$)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Output voltage (V)</th>
</tr>
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<tbody>
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<tr>
<td>10</td>
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<td>400 k</td>
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</tbody>
</table>

Important formulae introduced in this chapter

Open-loop voltage gain (pages 161 and 162):

$$A_{VOLI} = \frac{V_{OUT}}{V_{IN}}$$

$$A_{VOLI} = 20 \log_{10} \frac{V_{OUT}}{V_{IN}} \text{ dB}$$
8. Operational amplifiers

**Problems**

8.1 Sketch the circuit symbol for an operational amplifier. Label each of the connections.

8.2 List four characteristics associated with an 'ideal' operational amplifier.

8.3 An operational amplifier with negative feedback applied produces an output of 1.5 V when an input of 7.5 mV is present. Determine the closed-loop voltage gain.

8.4 Sketch the circuit of an inverting amplifier based on an operational amplifier. Label your circuit and identify the components that determine the closed-loop voltage gain.

8.5 Sketch the circuit of each of the following based on the use of operational amplifiers:
   (a) a comparator
   (b) a differentiator
   (c) an integrator.

8.6 An inverting amplifier is to be constructed having a mid-band voltage gain of 40, an input resistance of 5 kΩ and a frequency response extending from 20 Hz to 20 kHz. Devise a circuit and specify all component values required.

8.7 A summing amplifier with two inputs has $R_1 = 10 \, \text{kΩ}$, and $R_\text{IN}$ (for both inputs) of 2 kΩ. Determine the output voltage when one input is at $-2 \, \text{V}$ and the other is $+0.5 \, \text{V}$.

8.8 During measurements on an operational amplifier under open-loop conditions, an output voltage of 12 V is produced by an input voltage of 1 mV. Determine the open-loop voltage gain expressed in dB.

8.9 With the aid of a sketch, explain what is meant by the term ‘slew rate’. Why is this important?

Numerical answers to these problems appear on page 417.
Oscillators

Chapter summary
This chapter describes circuits that generate sine wave, square wave and triangular waveforms. These oscillator circuits form the basis of clocks and timing arrangements as well as signal and function generators.
**Positive feedback**

In Chapter 7 we showed how negative feedback can be applied to an amplifier to form the basis of a stage which has a precisely controlled gain. An alternative form of feedback, where the output is fed back in such a way as to reinforce the input (rather than to subtract from it), is known as positive feedback.

Fig. 9.1 shows the block diagram of an amplifier stage with positive feedback applied. Note that the amplifier provides a phase shift of 180° and the feedback network provides a further 180°. Thus the overall phase shift is 0°. The overall voltage gain, \( G \), is given by:

\[
G = \frac{V_{\text{out}}}{V_{\text{in}}}
\]

By applying Kirchhoff’s Voltage Law

\[ V_{n'} = V_n + \beta V_{\text{out}} \]

thus

\[ V_n = V_{n'} - \beta V_{\text{out}} \]

and

\[ V_{\text{out}} = A_v \times V_{\text{in}} \]

where \( A_v \) is the internal gain of the amplifier. Hence:

\[
\text{Overall gain, } G = \frac{A_v \times V_{n'}}{V_{n'} - \beta V_{\text{out}}} = \frac{A_v \times V_{n'}}{V_{n'} - \beta(A_v \times V_{\text{in}})}
\]

Thus, \( G = \frac{A_v}{1 - \beta A_v} \)

Now consider what will happen when the loop gain, \( \beta A_v \), approaches unity (i.e. when the loop gain is just less than 1). The denominator \( (1 - \beta A_v) \) will become close to zero. This will have the effect of *increasing* the overall gain, i.e. the overall gain with positive feedback applied will be greater than the gain without feedback.

It is worth illustrating this difficult concept using some practical figures. Assume that you have an amplifier with a gain of 9 and one-tenth of the output is fed back to the input (i.e. \( \beta = 0.1 \)). In this case the loop gain \( (\beta \times A_v) \) is 0.9.

With negative feedback applied (see Chapter 7) the overall voltage gain will be:

\[
G = \frac{A_v}{1+\beta A_v} = \frac{9}{1+(0.1 \times 9)} = \frac{9}{1+0.9} = \frac{9}{1.9} = 4.7
\]

With positive feedback applied the overall voltage gain will be:

\[
G = \frac{A_v}{1-\beta A_v} = \frac{10}{1-(0.1 \times 9)} = \frac{10}{1-0.9} = \frac{10}{0.1} = 90
\]

Now assume that you have an amplifier with a gain of 10 and, once again, one-tenth of the output is fed back to the input (i.e. \( \beta = 0.1 \)). In this example the loop gain \( (\beta \times A_v) \) is exactly 1.

With negative feedback applied (see Chapter 7) the overall voltage gain will be:

\[
G = \frac{A_v}{1+\beta A_v} = \frac{10}{1+(0.1 \times 10)} = \frac{10}{1+1} = \frac{10}{2} = 5
\]
9 Oscillators

With positive feedback applied the overall voltage gain will be:

\[ G = \frac{A_v}{1-\beta A_v} = \frac{10}{1-(0.1 \times 10)} = \frac{10}{1} = 10 \to \infty \]

This simple example shows that a loop gain of unity (or larger) will result in infinite gain and an amplifier which is unstable. In fact, the amplifier will oscillate since any disturbance will be amplified and result in an output.

Clearly, as far as an amplifier is concerned, positive feedback may have an undesirable effect – instead of reducing the overall gain the effect is that of reinforcing any signal present and the output can build up into continuous oscillation if the loop gain is 1 or greater. To put this another way, oscillator circuits can simply be thought of as amplifiers that generate an output signal without the need for an input!

**Conditions for oscillation**

From the foregoing we can deduce that the conditions for oscillation are:

(a) the feedback must be positive (i.e. the signal fed back must arrive back in-phase with the signal at the input);

(b) the overall loop voltage gain must be greater than 1 (i.e. the amplifier’s gain must be sufficient to overcome the losses associated with any frequency selective feedback network).

Hence, to create an oscillator we simply need an amplifier with sufficient gain to overcome the losses of the network that provide positive feedback. Assuming that the amplifier provides 180° phase shift, the frequency of oscillation will be that at which there is 180° phase shift in the feedback network.

A number of circuits can be used to provide 180° phase shift, one of the simplest being a three-stage C–R ladder network that we shall meet next. Alternatively, if the amplifier produces 0° phase shift, the circuit will oscillate at the frequency at which the feedback network produces 0° phase shift. In both cases, the essential point is that the feedback should be positive so that the output signal arrives back at the input in such a sense as to reinforce the original signal.

**Ladder network oscillator**

A simple phase-shift oscillator based on a three-stage C–R ladder network is shown in Fig. 9.2. TR1 operates as a conventional common-emitter amplifier stage with \( R_1 \) and \( R_2 \) providing base bias potential and \( R_3 \) and \( C_1 \) providing emitter stabilization.

The total phase shift provided by the C–R ladder network (connected between collector and base) is 180° at the frequency of oscillation. The transistor provides the other 180° phase shift in order to realize an overall phase shift of 360° or 0° (note that these are the same).

The frequency of oscillation of the circuit shown in Fig. 9.2 is given by:

\[ f = \frac{1}{2\pi \sqrt{BCR}} \]

The loss associated with the ladder network is 29, thus the amplifier must provide a gain of at least 29 in order for the circuit to oscillate. In practice this is easily achieved with a single transistor.

**Example 9.1**

Determine the frequency of oscillation of a three-stage ladder network oscillator in which \( C = 10 \, \text{nF} \) and \( R = 10 \, \text{k} \Omega \).
The frequency at which the phase shift will be zero is given by:

\[ f = \frac{1}{2\pi \sqrt{CR}} \]

When \( R_1 = R_2 \) and \( C_1 = C_2 \) the frequency at which the phase shift will be zero will be given by:

\[ f = \frac{1}{2\pi \sqrt{C^2R^2}} = \frac{1}{2\pi CR} \]

where \( R = R_1 = R_2 \) and \( C = C_1 = C_2 \).

**Example 9.2**

Fig. 9.4 shows the circuit of a Wien bridge oscillator based on an operational amplifier.

If \( C_1 = C_2 = 100 \text{ nF} \), determine the output frequencies produced by this arrangement (a) when \( R_1 = R_2 = 1 \text{ k}\Omega \) and (b) when \( R_1 = R_2 = 6 \text{ k}\Omega \).

**Solution**

(a) When \( R_1 = R_2 = 1 \text{ k}\Omega \)

\[ f = \frac{1}{2\pi CR} \]

where \( R = R_1 = R_1 \) and \( C = C_1 = C_2 \).

Thus

\[ f = \frac{10^4}{6.28 \times 100 \times 10^{-9} \times 1 \times 10^3} = \frac{10^4}{6.28} = 1.59 \text{ kHz} \]

(b) When \( R_1 = R_2 = 6 \text{ k}\Omega \)

\[ f = \frac{1}{2\pi \sqrt{C^2R^2}} \]

\[ f = \frac{1}{2\pi \sqrt{600 \times 10^6}} = \frac{1}{2\pi \times 2.45 \times 10^3} = 647 \text{ Hz} \]

**Wien bridge oscillator**

An alternative approach to providing the phase shift required is the use of a Wien bridge network (Fig. 9.3). Like the C–R ladder, this network provides a phase shift which varies with frequency. The input signal is applied to A and B while the output is taken from C and D. At one particular frequency, the phase shift produced by the network will be exactly zero (i.e. the input and output signals will be in-phase). If we connect the network to an amplifier producing 0° phase shift which has sufficient gain to overcome the losses of the Wien bridge, oscillation will result.

The minimum amplifier gain required to sustain oscillation is given by:

\[ A_c = 1 + \frac{R_2}{C_2 \cdot R_1} \]

In most cases, \( C_1 = C_2 \) and \( R_1 = R_2 \), hence the minimum amplifier gain will be 3.
9 Oscillators

(b) When \( R_1 = R_1 = 6 \, \text{k}\Omega \)

\[
f = \frac{1}{2\pi CR}
\]

where \( R = R_1 = R_1 \) and \( C = C_1 = C_2 \).

Thus

\[
f = \frac{1}{6.28 \times 100 \times 10^{-9} \times 6 \times 10^3}
\]

\[
f = \frac{10^4}{37.68} = 265 \, \text{Hz}
\]

Multivibrators

There are many occasions when we require a square wave output from an oscillator rather than a sine wave output. Multivibrators are a family of oscillator circuits that produce output waveforms consisting of one or more rectangular pulses. The term ‘multivibrator’ simply originates from the fact that this type of waveform is rich in harmonics (i.e. ‘multiple vibrations’).

Multivibrators use regenerative (i.e. positive) feedback; the active devices present within the oscillator circuit being operated as switches, being alternately cut-off and driven into saturation.

The principal types of multivibrator are:

(a) **astable multivibrators** that provide a continuous train of pulses (these are sometimes also referred to as free-running multivibrators);

(b) **monostable multivibrators** that produce a single output pulse (they have one stable state and are thus sometimes also referred to as ‘one-shot’);

(c) **bistable multivibrators** that have two stable states and require a trigger pulse or control signal to change from one state to another.

The astable multivibrator

Fig. 9.6 shows a classic form of astable multivibrator based on two transistors. Fig. 9.7 shows how this circuit can be redrawn in an arrangement that more closely resembles a two-stage common-emitter amplifier with its output connected back to its input. In Fig. 9.6, the values of the base resistors, \( R_3 \) and \( R_4 \), are such that the sufficient base current will be available to completely saturate the respective transistor. The values of the collector load resistors, \( R_1 \) and \( R_2 \), are very much smaller than \( R_3 \) and \( R_4 \). When power is first applied to the circuit, assume that TR2 saturates before TR1 when the power is first applied (in practice one transistor would always saturate before the other due to variations in component tolerances and transistor parameters).

As TR2 saturates, its collector voltage will fall rapidly from \(+V_{cc}\) to 0 V. This drop in voltage will be transferred to the base of TR1 via \( C_1 \). This negative-going voltage will ensure that TR1 is initially placed in the non-conducting state. As long as TR1 remains cut-off, TR2 will continue to be saturated. During this time, \( C_1 \) will charge via \( R_4 \) and TR1’s base voltage will rise exponentially.
Oscillators

TR2 is high (T2) will be determined by the time constant, $R_3 \times C_1$.

The following approximate relationships apply:

$$T_1 = 0.7 \frac{C_2}{R_4} \quad \text{and} \quad T_2 = 0.7 \frac{C_1}{R_3}$$

Since one complete cycle of the output occurs in a time, $T = T_1 + T_2$, the periodic time of the output is given by:

$$T = 0.7 \left( C_2 R_4 + C_1 R_3 \right)$$

Finally, we often require a symmetrical square wave output where $T_1 = T_2$. To obtain such an output, we should make $R_3 = R_4$ and $C_1 = C_2$, in which case the periodic time of the output will be given by:

$$T = 1.4 CR$$

where $C = C_1 = C_2$ and $R = R_3 = R_4$. Waveforms for the astable oscillator are shown in Fig. 9.8.

Example 9.3

The astable multivibrator in Fig. 9.6 is required to produce a square wave output at 1 kHz. Determine suitable values for $R_3$ and $R_4$ if $C_1$ and $C_2$ are both 10 nF.

Solution

Since a square wave is required and $C_1$ and $C_2$ have identical values, $R_3$ must be made equal to $R_4$. Now:

$$T = \frac{1}{f} = \frac{1}{1 \times 10^3} = 1 \times 10^{-3} \text{ s}$$

Re-arranging $T = 1.4 CR$ to make $R$ the subject gives:

$$R = \frac{T}{1.4C} = \frac{1 \times 10^{-3}}{1.4 \times 10^6} = \frac{1 \times 10^6}{14} = 0.071 \times 10^6$$

hence

$$R = 71 \times 10^3 = 71 \text{k}\Omega$$

Other forms of astable oscillator

Fig. 9.9 shows the circuit diagram of an alternative form of astable oscillator which produces a triangular output waveform. Operational amplifier IC1 forms an integrating stage while IC2 is connected with positive feedback to ensure that oscillation takes place.
9 Oscillators

Assume that the output from IC2 is initially at, or near, $+V_{CC}$ and capacitor, $C$, is uncharged. The voltage at the output of IC2 will be passed, via $R$, to IC1. Capacitor, $C$, will start to charge and the output voltage of IC1 will begin to fall.

Eventually, the output voltage will have fallen to a value that causes the polarity of the voltage at the non-inverting input of IC2 to change from positive to negative. At this point, the output of IC2 will rapidly fall to $-V_{CC}$. Again, this voltage will be passed, via $R$, to IC1. Capacitor $C$ will then start to charge in the other direction and the output voltage of IC1 will begin to rise.

Some time later, the output voltage will have risen to a value that causes the polarity of the non-inverting input of IC2 to revert to its original (positive) state and the cycle will continue indefinitely.

The upper threshold voltage (i.e. the maximum positive value for $V_{out}$) will be given by:

$$V_{UT} = V_{CC} \times \left( \frac{R1}{R2} \right)$$

The lower threshold voltage (i.e. the maximum negative value for $V_{out}$) will be given by:

$$V_{LT} = -V_{CC} \times \left( \frac{R1}{R2} \right)$$

**Single-stage astable oscillator**

A simple form of astable oscillator that produces a square wave output can be built using just one operational amplifier, as shown in Fig. 9.10. The circuit employs positive feedback with the output fed back to the non-inverting input via the potential divider formed by $R1$ and $R2$. This circuit can make a very simple square wave source with a frequency that can be made adjustable by replacing $R$ with a variable or preset resistor.

Assume that $C$ is initially uncharged and the voltage at the inverting input is slightly less than the voltage at the non-inverting input. The output voltage will rise rapidly to $+V_{CC}$ and the voltage at the inverting input will begin to rise exponentially as capacitor $C$ charges through $R$.

Eventually the voltage at the inverting input will have reached a value that causes the voltage at the inverting input to exceed that present at the non-inverting input. At this point, the output voltage will rapidly fall to $-V_{CC}$. Capacitor $C$ will then start to charge in the other direction and the voltage at the inverting input will begin to fall exponentially.

Eventually, the voltage at the inverting input will have reached a value that causes the voltage at the inverting input to be less than that present at the non-inverting input. At this point, the output voltage will rise rapidly to $+V_{CC}$ once again and the cycle will continue indefinitely.

The upper threshold voltage (i.e. the maximum positive value for the voltage at the inverting input) will be given by:

$$V_{UT} = V_{CC} \times \left( \frac{R2}{R1+R2} \right)$$
The lower threshold voltage (i.e. the maximum negative value for the voltage at the inverting input) will be given by:

\[ V_{LT} = -V_{CC} \times \left( \frac{R_2}{R_1 + R_2} \right) \]

Finally, the time for one complete cycle of the output waveform produced by the astable oscillator is given by:

\[ T = 2CR \ln \left( 1 + 2 \left( \frac{R_2}{R_1} \right) \right) \]

**Crystal controlled oscillators**

A requirement of some oscillators is that they accurately maintain an exact frequency of oscillation. In such cases, a quartz crystal can be used as the frequency determining element. The quartz crystal (a thin slice of quartz in a hermetically sealed enclosure, see Fig. 9.11) vibrates whenever a potential difference is applied across its faces (this phenomenon is known as the piezoelectric effect). The frequency of oscillation is determined by the crystal’s ‘cut’ and physical size.

Most quartz crystals can be expected to stabilize the frequency of oscillation of a circuit to within a few parts in a million. Crystals can be manufactured for operation in fundamental mode over a frequency range extending from 100 kHz to around 20 MHz and for overtone operation from 20 MHz to well over 100 MHz.

Fig. 9.12 shows a simple crystal oscillator circuit in which the crystal provides feedback from the drain to the source of a junction gate FET.

**Practical oscillator circuits**

Fig. 9.13 shows a practical sine wave oscillator based on a three-stage C–R ladder network. The circuit provides an output of approximately 1 V peak–peak at 1.97 kHz.

A practical Wien bridge oscillator is shown in Fig. 9.14. This circuit produces a sine wave output at 16 Hz. The output frequency can easily be varied by making \( R_1 \) and \( R_2 \) a 10 kΩ dual-gang potentiometer and connecting a fixed resistor of 680 Ω in series with each. In order to adjust the loop gain for an optimum sine wave output it may be necessary to make \( R_3/R_4 \) adjustable. One way of doing this is to replace both components with a 10 kΩ multi-turn potentiometer with the sliding contact taken to the inverting input of IC1.

---

**Figure 9.12** A simple JFET oscillator

**Figure 9.13** A practical sine wave oscillator based on a phase shift ladder network
9 Oscillators

An astable multivibrator is shown in Fig. 9.15. This circuit produces a square wave output of 5 V peak–peak at approximately 690 Hz.

A triangle wave generator is shown in Fig. 9.16. This circuit produces a symmetrical triangular output waveform at approximately 8 Hz. If desired, a simultaneous square wave output can be derived from the output of IC2. The circuit requires symmetrical supply voltage rails (not shown in Fig. 9.14) of between ±9 V and ±15 V.

Fig. 9.17 shows a single-stage astable oscillator. This circuit produces a square wave output at approximately 13 Hz.

Finally, Fig. 9.18 shows a high-frequency crystal oscillator that produces an output of approximately 1 V peak–peak at 4 MHz. The precise frequency of operation depends upon the quartz crystal employed (the circuit will operate with fundamental mode crystals in the range 2 MHz to about 12 MHz).

Practical investigation

Objective

To investigate a simple operational amplifier astable oscillator.
Components and test equipment

Breadboard, oscilloscope, ±9 V d.c. power supply (or two 9 V batteries), 741CN (or similar operational amplifier), 10 n, 22 n, 47 n and 100 n capacitors, resistors of 100 kΩ, 1 kΩ and 680 Ω, 5% 0.25 W, test leads, connecting wire.

Procedure

Connect the circuit shown in Fig. 9.19 with $C = 47$ nF. Set the oscilloscope timebase to the 2 ms/cm range and Y-attenuator to 1 V/cm. Adjust the oscilloscope so that it triggers on a positive edge and display the output waveform produced by the oscillator. Make a sketch of the waveform using the graph layout shown in Fig. 9.20.

Measure and record (see Table 9.1) the time for one complete cycle of the output. Repeat this measurement with $C = 10$ nF, 22 nF and 100 nF.

Calculations

For each value of $C$, calculate the periodic time of the oscillator’s output and compare this with the measured values.

Conclusion

Comment on the performance of the astable oscillator. Is this what you would expect? Do the measured values agree with those obtained by calculation? If not, suggest reasons for any differences. Suggest typical applications for the circuit.

Symbol introduced in this chapter

Quartz crystal (or piezo resonator)

Figure 9.19 Astable oscillator circuit used in the Practical investigation

Figure 9.20 Graph layout for sketching the output waveform produced by the astable oscillator

Table 9.1 Table of results and calculated values

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<th>$C$</th>
<th>Measured periodic time</th>
<th>Calculated periodic time</th>
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<td></td>
<td></td>
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<tr>
<td>100 nF</td>
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</tbody>
</table>

Important formulae introduced in this chapter

Gain with positive feedback (page 175):

$$G = \frac{A_v}{1 - \beta A_v}$$
9 Oscillators

Loop gain:
(page 175)
\[ L = \beta A_v \]

Output frequency of a three-stage C–R ladder network oscillator:
(page 176)
\[ f = \frac{1}{2\pi \sqrt{RC}} \]

Output frequency of a Wien bridge oscillator:
(page 177)
\[ f = \frac{1}{2\pi CR} \]

Time for which a multivibrator output is ‘high’:
(page 179)
\[ T_1 = 0.7 \ C \ 2 \ R_4 \]

Time for which a multivibrator output is ‘low’:
(page 179)
\[ T_2 = 0.7 \ C \ 1 \ R_3 \]

Periodic time for the output of a square wave multivibrator:
(page 179)
\[ T = 0.7 \ (C_2 \ R_4 + C_1 \ R_3) \]

when \( C = C_1 = C_2 \) and \( R = R_3 = R_4 \)
\[ T = 1.4 \ C \ R \]

Periodic time for the output of a single-stage astable oscillator:
(page 181)
\[ T = 2\pi R \ln \left(1 + 2 \left(\frac{R_2}{R_1}\right)\right) \]

Problems

9.1 An amplifier with a gain of 8 has 10% of its output fed back to the input. Determine the gain of the stage (a) with negative feedback, (b) with positive feedback.

9.2 A phase-shift oscillator is to operate with an output at 1 kHz. If the oscillator is based on a three-stage ladder network, determine the required values of resistance if three capacitors of 10 nF are to be used.

9.3 A Wien bridge oscillator is based on the circuit shown in Fig. 9.4 but \( R_1 \) and \( R_2 \) are replaced by a dual-gang potentiometer. If \( C_1 = C_2 = 22 \ \text{nF} \), determine the values of \( R_1 \) and \( R_2 \) required to produce an output at exactly 400 Hz.

9.4 Determine the peak–peak voltage developed across \( C_1 \) in the oscillator circuit shown in Fig. 9.22.

9.5 Determine the periodic time and frequency of the output signal produced by the oscillator circuit shown in Fig. 9.22.

9.6 An astable multivibrator circuit is required to produce an asymmetrical rectangular output which has a period of 4 ms and is to be ‘high’ for 1 ms and ‘low’ for 3 ms. If the timing capacitors are both to be 100 nF, determine the values of the two timing resistors required.

9.7 Explain, briefly, how the astable multivibrator shown in Fig. 9.23 operates. Illustrate your answer using a waveform sketch.
9.8 Determine the output frequency of the signal produced by the circuit shown in Fig. 9.23.

9.9 Explain, briefly, how the Wien bridge oscillator shown in Fig. 9.24 operates. What factors affect the choice of values for \( R_3 \) and \( R_4 \)?

9.10 Determine the output frequency of the signal produced by the circuit shown in Fig. 9.24.

9.11 Sketch the circuit of an oscillator that will produce a triangular waveform output. Explain briefly how the circuit operates and suggest a means of varying the output frequency over a limited range.

9.12 Distinguish between the following types of multivibrator circuit:
   (a) astable multivibrators, (b) monostable multivibrators, (c) bistable multivibrators.

9.13 Derive an expression (in terms of \( R_3 \) and \( R_4 \)) for the minimum value of voltage gain required to produce oscillation in the circuit shown in Fig. 9.25.

9.14 Design an oscillator circuit that will generate the output waveform shown in Fig. 9.26. Sketch a circuit diagram for the oscillator and specify all component values (including supply voltage). Give reasons for your choice of oscillator circuit.

9.15 Design an oscillator circuit that will generate the output waveform shown in Fig. 9.27. Sketch a circuit diagram for the oscillator and specify all component values.
9. Oscillators

Figure 9.28  See Question 9.16

values (including supply voltage). Give reasons for your choice of oscillator circuit.

9.16  Design an oscillator circuit that will generate the output waveform shown in Fig. 9.28. Sketch a circuit diagram for the oscillator and specify all component values (including supply voltage). Give reasons for your choice of oscillator circuit.

9.17  Briefly explain the term ‘piezoelectric effect’.

9.18  Sketch the circuit diagram of a simple single-stage crystal oscillator and explain the advantages of using a quartz crystal as the frequency determining element.

Answers to these problems appear on page 417.
Chapter summary

This chapter introduces electronic circuits and devices that are associated with digital rather than analogue circuitry. These logic circuits are used extensively in digital systems and form the basis of clocks, counters, shift registers and timers.

The chapter starts by introducing the basic logic functions (AND, OR, NAND, NOR, etc.) together with the symbols and truth tables that describe the operation of the most common logic gates. We then show how these gates can be used in simple combinational logic circuits before moving on to introduce bistable devices, counters and shift registers. The chapter concludes with a brief introduction to the two principal technologies used in modern digital logic circuits, TTL and CMOS.
### Logic functions

Electronic logic circuits can be used to make simple decisions like:

*If dark then put on the light.*

and

*If temperature is less then 20 °C then connect the supply to the heater.*

They can also be used to make more complex decisions like:

*If ‘hour’ is greater than 11 and ‘24 hour clock’ is not selected then display message ‘pm’.*

All of these logical statements are similar in form. The first two are essentially:

*If {condition} then {action}.*

while the third is a compound statement of the form:

*If {condition 1} and not {condition 2} then {action}.*

Both of these statements can be readily implemented using straightforward electronic circuits. Because this circuitry is based on discrete states and since the behaviour of the circuits can be described by a set of logical statements, it is referred to as **digital logic**.

### Switch and lamp logic

In the simple circuit shown in Fig. 10.1 a battery is connected to a lamp via a switch. There are two possible states for the switch, open and closed, but the lamp will only operate when the switch is closed. We can summarize this using Table 10.1.

Since the switch can only be in one of the two states (i.e. open or closed) at any given time, the open and closed conditions are mutually exclusive. Furthermore, since the switch cannot exist in any other state than completely open or completely closed (i.e. there is no intermediate or half-open state) the circuit uses binary or ‘two-state’ logic. The logical state of the switch can be represented by the **binary digits**, 0 and 1. For example, if logical 0 is synonymous with open (or ‘off’) and logical 1 is equivalent to closed (or ‘on’), then:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Switch</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open</td>
<td>No light produced</td>
</tr>
<tr>
<td>2</td>
<td>Closed</td>
<td>Light produced</td>
</tr>
</tbody>
</table>

Switch open (off) = 0  
Switch closed (on) = 1

We can now rewrite the **truth table** in terms of the binary states as shown in Fig. 10.2 where:

No light (off) = 0  
Light (on) = 1

### AND logic

Now consider the circuit with two switches shown in Fig. 10.3. Here, the lamp will only operate when switch A is closed and switch B is closed. However, let’s look at the operation of the circuit in a little more detail.

Since there are two switches (A and B) and there are two possible states for each switch (open or closed), there is a total of four possible conditions for the circuit. We summarize these conditions in Table 10.2.

Since each switch can only be in one of the two states (i.e. open or closed) at any given time, the open and closed conditions are mutually exclusive. Furthermore, since the switches cannot exist in any other state than completely open or completely closed (i.e. there are no intermediate states) the circuit uses **binary logic**. We can thus represent the logical states of the two switches by the binary digits, 0 and 1.

Once again, if we adopt the convention that an open switch can be represented by 0 and a closed switch by 1, we can rewrite the truth table.
10 Logic circuits

OR logic

Fig. 10.5 shows another circuit with two switches. This circuit differs from that shown in Fig. 10.3 by virtue of the fact that the two switches are connected in parallel rather than in series. In this case the lamp will operate when either of the two switches is closed. As before, there is a total of four possible conditions for the circuit. We summarize these conditions in Table 10.3.

Once again, adopting the convention that an open switch can be represented by 0 and a closed switch by 1, we can rewrite the truth table in terms of the binary states as shown in Fig. 10.6.

Example 10.1

Fig. 10.7 shows a simple switching circuit. Describe the logical state of switches A, B and C in order to operate the lamp. Illustrate your answer with a truth table.

Table 10.3 Simple OR switching logic

<table>
<thead>
<tr>
<th>Condition</th>
<th>Switch A</th>
<th>Switch B</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open</td>
<td>Open</td>
<td>No light produced</td>
</tr>
<tr>
<td>2</td>
<td>Open</td>
<td>Closed</td>
<td>No light produced</td>
</tr>
<tr>
<td>3</td>
<td>Closed</td>
<td>Open</td>
<td>No light produced</td>
</tr>
<tr>
<td>4</td>
<td>Closed</td>
<td>Closed</td>
<td>Light produced</td>
</tr>
</tbody>
</table>

in terms of the binary states shown in Fig. 10.4 where:

No light (off) = 0
Light (on) = 1

Figure 10.2 Truth table for the switch and lamp

Figure 10.3 AND switch and lamp logic

Table 10.2 Simple AND switching logic

<table>
<thead>
<tr>
<th>Condition</th>
<th>Switch A</th>
<th>Switch B</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open</td>
<td>Open</td>
<td>No light produced</td>
</tr>
<tr>
<td>2</td>
<td>Open</td>
<td>Closed</td>
<td>No light produced</td>
</tr>
<tr>
<td>3</td>
<td>Closed</td>
<td>Open</td>
<td>No light produced</td>
</tr>
<tr>
<td>4</td>
<td>Closed</td>
<td>Closed</td>
<td>Light produced</td>
</tr>
</tbody>
</table>

Figure 10.4 Truth table for the switch and lamp

Figure 10.5 OR switch and lamp logic

Figure 10.6 Truth table for OR logic

Figure 10.7 See Example 10.1
10 Logic circuits

Solution
In order to operate the lamp, switch A and either switch B or switch C must be operated. The truth table is shown in Fig. 10.8.

Logic gates
Logic gates are circuits designed to produce the basic logic functions, AND, OR, etc. These circuits are designed to be interconnected into larger, more complex, logic circuit arrangements. Since these circuits form the basic building blocks of all digital systems, we have summarized the action of each of the gates in the next section. For each gate we have included its British Standard (BS) symbol together with its American Standard (MIL/ANSI) symbol. We have also included the truth tables and Boolean expressions (using ‘+’ to denote OR, ‘·’ to denote AND, and ‘¬’ to denote NOT). Note that, while inverters and buffers each have only one input, exclusive-OR gates have two inputs and the other basic gates (AND, OR, NAND and NOR) are commonly available with up to eight inputs.

Buffers (Fig. 10.9)
Buffers do not affect the logical state of a digital signal (i.e. a logic 1 input results in a logic 1 output whereas a logic 0 input results in a logic 0 output). Buffers are normally used to provide extra current drive at the output but can also be used to regularize the logic levels present at an interface. The Boolean expression for the output, \( Y \), of a buffer with an input, \( X \), is:
\[ Y = X \]

Inverters (Fig. 10.10)
Inverters are used to complement the logical state (i.e. a logic 1 input results in a logic 0 output and vice versa). Inverters also provide extra current drive and, like buffers, are used in interfacing applications where they provide a means of regularizing logic levels present at the input or output of a digital system. The Boolean expression for the output, \( Y \), of a buffer with an input, \( X \), is:
\[ Y = \overline{X} \]

Figure 10.8 See Example 10.1

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 10.9 Symbols and truth table for a buffer

Figure 10.10 Symbols and truth table for an inverter

AND gates (Fig. 10.11)
AND gates will only produce a logic 1 output when all inputs are simultaneously at logic 1. Any other input combination results in a logic 0 output. The Boolean expression for the output, \( Y \), of an AND gate with inputs, \( A \) and \( B \), is:
\[ Y = A \cdot B \]

OR gates (Fig. 10.12)
OR gates will produce a logic 1 output whenever any one, or more, inputs are at logic 1. Putting this another way, an OR gate will only produce a logic 0 output whenever all of its inputs are simultaneously at logic 0. The Boolean expression for the output, \( Y \), of an OR gate with inputs \( A \) and \( B \) is:
\[ Y = A + B \]
Combinational logic

By using a standard range of logic levels (i.e. voltage levels used to represent the logic 1 and logic 0 states) logic circuits can be combined in order to solve complex logic functions.

Example 10.2

A logic circuit is to be constructed that will produce a logic 1 output whenever two or more of its three inputs are at logic 1.

Solution

This circuit could be more aptly referred to as a majority vote circuit. Its truth table is shown in Fig. 10.16. Fig. 10.17 shows the logic circuitry required.

Example 10.3

Show how an arrangement of basic logic gates (AND, OR and NOT) can be used to produce the exclusive-OR function.
10 Logic circuits

R-S bistables

The simplest form of bistable is the R-S bistable. This device has two inputs, SET and RESET, and complementary outputs, Q and \( \bar{Q} \). A logic 1 applied to the SET input will cause the Q output to become (or remain at) logic 1 while a logic 1 applied to the RESET input will cause the Q output to become (or remain at) logic 0. In either case, the bistable will remain in its SET or RESET state until an input is applied in such a sense as to change the state.

Two simple forms of R-S bistable based on cross-coupled logic gates are shown in Fig. 10.19. Fig. 10.19(a) is based on NAND gates while Fig. 10.19(b) is based on NOR gates.

Solution

In order to solve this problem, consider the Boolean expression for the exclusive-OR function:

\[ Y = A \cdot \bar{B} + B \cdot \bar{A} \]

This expression takes the form:

\[ Y = P + Q \text{ where } P = A \cdot \bar{B} \text{ and } Q = B \cdot \bar{A} \]

\( A \cdot \bar{B} \) and \( B \cdot \bar{A} \) can be obtained using two two-input AND gates and the result (i.e. \( P \) and \( Q \)) can then be applied to an OR gate with two inputs. \( \bar{A} \) and \( \bar{B} \) can be produced using inverters. The complete solution is shown in Fig. 10.18.

Bistables

The output of a bistable has two stable states (logic 0 or logic 1) and, once set in one or other of these states, the device will remain at a particular logic level for an indefinite period until reset. A bistable thus constitutes a simple form of ‘memory cell’ because it will remain in its latched state (whether set or reset) until a signal is applied to it in order to change its state (or until the supply is disconnected).
The simple cross-coupled logic gate bistable has a number of serious shortcomings (consider what would happen if a logic 1 was simultaneously present on both the SET and RESET inputs) and practical forms of bistable make use of much improved purpose-designed logic circuits such as D-type and J-K bistables.

**D-type bistables**

The D-type bistable has two inputs: D (standing variously for ‘data’ or ‘delay’) and CLOCK (CLK). The data input (logic 0 or logic 1) is clocked into the bistable such that the output state only changes when the clock changes state. Operation is thus said to be synchronous. Additional subsidiary inputs (which are invariably active low) are provided which can be used to directly set or reset the bistable. These are usually called PRESET (PR) and CLEAR (CLR). D-type bistables are used both as latches (a simple form of memory) and as binary dividers. The simple circuit arrangement in Fig. 10.20 together with the timing diagram shown in Fig. 10.21 illustrate the operation of D-type bistables.

![Figure 10.20](image1.png)  
Figure 10.20  D-type bistable operation

![Figure 10.21](image2.png)  
Figure 10.21  Timing diagram for the D-type bistable

**Table 10.4**  Input and output states for a J-K bistable (PRESET and CLEAR inputs)

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESET</td>
<td>CLEAR</td>
<td>Q&lt;sub&gt;N+1&lt;/sub&gt;</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>? Indeterminate</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0 (i.e. Q is reset) regardless of the clock state</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1 (i.e. Q is set) regardless of the clock state</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>See below Enables clocked operation (refer to Table 10.5)</td>
</tr>
</tbody>
</table>

Note: The preset and clear inputs operate regardless of the clock

**Table 10.5**  Input and output states for a J-K bistable (clocked operation)

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>K</td>
<td>Q&lt;sub&gt;N+1&lt;/sub&gt;</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>Q&lt;sub&gt;N&lt;/sub&gt; No change in state of the Q output on the next clock transition</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0 (i.e. Q is reset) on the next clock transition</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1 (i.e. Q is set) on the next clock transition</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Q&lt;sub&gt;N&lt;/sub&gt; Q output changes to the opposite state on the next clock transition</td>
</tr>
</tbody>
</table>

Note: Q<sub>N+1</sub> means ‘Q after next clock transition’ while Q<sub>N</sub> means ‘Q in whatever state it was before’

**J-K bistables**

J-K bistables have two clocked inputs (J and K), two direct inputs (PRESET and CLEAR), a CLOCK (CK) input, and outputs (Q and Q̅). As with R-S bistables, the two outputs are complementary (i.e. when one is 0 the other is 1, and vice versa). Similarly, the PRESET and CLEAR inputs are invariably both active low (i.e. a 0 on the PRESET input will set the Q output to 1 whereas a 0 on the CLEAR input will set the Q output to 0).

Tables 10.4 and 10.5 summarize the operation of a J-K bistable, respectively, for the PRESET and CLEAR inputs and for clocked operation.
J-K bistables are the most sophisticated and flexible of the bistable types and they can be configured in various ways including binary dividers, shift registers and latches.

Fig. 10.22 shows the arrangement of a four-stage binary counter based on J-K bistables. The timing diagram for this circuit is shown in Fig. 10.23. Each stage successively divides the clock input signal by a factor of two. Note that a logic 1 input is transferred to the respective Q-output on the falling edge of the clock pulse and all J and K inputs must be taken to logic 1 to enable binary counting.

Fig. 10.24 shows the arrangement of a four-stage shift register based on J-K bistables. The timing diagram for this circuit is shown in Fig. 10.25. Note that each stage successively feeds data to the next stage. Note that all data transfer occurs on the falling edge of the clock pulse.

**Example 10.4**

A logic arrangement has to be designed so that it produces the pulse train shown in Fig. 10.27. Devise a logic circuit arrangement that will generate this pulse train from a regular square wave input.

**Solution**

A two-stage binary divider (based on J-K bistables) can be used together with a two-input AND gate as shown in Fig. 10.26. The waveforms for this logic arrangement are shown in Fig. 10.28.
Figure 10.24  Four-stage shift register using J-K bistables

Figure 10.25  Timing diagram for the four-stage shift register shown in Fig. 10.24

Figure 10.26  See Example 10.4

Figure 10.27  See Example 10.4

Figure 10.28  Waveforms for the logic arrangement shown in Fig. 10.26

Figure 10.29  See Example 10.4
10 Logic circuits

Integrated circuit logic devices

The task of realizing a complex logic circuit is made simple with the aid of digital integrated circuits. Such devices are classified according to the semiconductor technology used in their fabrication (the logic family to which a device belongs is largely instrumental in determining its operational characteristics, such as power consumption, speed and immunity to noise).

The relative size of a digital integrated circuit (in terms of the number of active devices that it contains) is often referred to as its scale of integration and the terminology in Table 10.6 is commonly used.

The two basic logic families are CMOS (complementary metal oxide semiconductor) and TTL (transistor transistor logic). Each of these families is then further sub-divided. Representative circuits for a two-input AND gate in both technologies are shown in Figs 10.29 and 10.30.

Table 10.6 Scale of integration

<table>
<thead>
<tr>
<th>Scale of integration</th>
<th>Abbreviation</th>
<th>Number of logic gates*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>SSI</td>
<td>1 to 10</td>
</tr>
<tr>
<td>Medium</td>
<td>MSI</td>
<td>10 to 100</td>
</tr>
<tr>
<td>Large</td>
<td>LSI</td>
<td>100 to 1,000</td>
</tr>
<tr>
<td>Very large</td>
<td>VLSI</td>
<td>1,000 to 10,000</td>
</tr>
<tr>
<td>Super large</td>
<td>SLSI</td>
<td>10,000 to 100,000</td>
</tr>
</tbody>
</table>

* or active circuitry of equivalent complexity

Example 10.5

Identify each of the following integrated circuits:

(i) 4001UBE;
(ii) 74LS14.

Solution

Integrated circuit (i) is an improved (unbuffered) version of the CMOS 4001 device. Integrated circuit (ii) is a low-power Schottky version of the TTL 7414 device.

Date codes

It is also worth noting that the vast majority of logic devices and other digital integrated circuits are marked with a four-digit date code. The code often appears alongside or below the device code. The first two digits of this code give the year of manufacture while the last two digits specify the week of manufacture.

Example 10.6

An integrated circuit is marked ‘4050B 9832’. What type of device is it and when was it manufactured?
Logic levels

Logic levels are simply the range of voltages used to represent the logic states 0 and 1. The logic levels for CMOS differ markedly from those associated with TTL. In particular, CMOS logic levels are relative to the supply voltage used while the logic levels associated with TTL devices tend to be absolute (see Table 10.9).

Noise margin

The noise margin of a logic device is a measure of its ability to reject noise and spurious signals; the larger the noise margin the better is its ability to perform in an environment in which noise is present. Noise margin is defined as the difference between the minimum values of high-state output and high-state input voltage and the maximum values of low-state output and low-state input voltage. Hence:

\[
\text{Noise margin} = V_{OH(\text{MIN})} - V_{IH(\text{MIN})}
\]

or

\[
\text{Noise margin} = V_{OL(\text{MAX})} - V_{OH(\text{MAX})}
\]

Table 10.9 Logic levels for CMOS and TTL logic devices

<table>
<thead>
<tr>
<th>Condition</th>
<th>CMOS</th>
<th>TTL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic 0</td>
<td>Less than 1/3(V_{dd})</td>
<td>More than 2 V</td>
</tr>
<tr>
<td>Logic 1</td>
<td>More than 2/3(V_{dd})</td>
<td>Less than 0.8 V</td>
</tr>
<tr>
<td>Indeterminate</td>
<td>Between 1/3(V_{dd}) and 2/3(V_{dd})</td>
<td>Between 0.8 V and 2 V</td>
</tr>
</tbody>
</table>

Note: \(V_{dd}\) is the positive supply associated with CMOS devices.
where $V_{	ext{OH(MIN)}}$ is the minimum value of high-state (logic 1) output voltage, $V_{	ext{IH(MIN)}}$ is the minimum value of high-state (logic 1) input voltage, $V_{	ext{OL(MAX)}}$ is the maximum value of low-state (logic 0) output voltage, and $V_{	ext{OH(MAX)}}$ is the maximum value of low-state (logic 0) input voltage.

The noise margin for standard 7400 series TTL is typically 400 mV while that for CMOS is $1/3V_{	ext{DD}}$, as shown in Fig. 10.31.

Table 10.10 compares the more important characteristics of common members of the TTL family with their buffered CMOS logic counterparts. Finally, Fig. 10.32 shows the packages and pin connections for two common logic devices, the 74LS00 (quad two-input NAND gate) and the 4001UBE (quad two-input NOR gate).

Example 10.7
Show how a 4001UBE device (see Fig. 10.32) can be connected to form a simple cross-coupled bistable. Sketch a circuit diagram showing pin connections and include LEDs that will indicate the output state of the bistable.

Solution
See Practical investigation below. Note that only two of the four logic gates have been used.

Practical investigation
Objective
To investigate the operation of a simple bistable based on cross-coupled NOR gates.

Components and test equipment
Breadboard, 9 V d.c. power supply (or a 9 V battery), 4001BE quad two-input buffered CMOS NOR gate, red and green LEDs, operational amplifier), two 1 kΩ and two 47 kΩ 5% 0.25 W resistors, test leads, connecting wire.

Table 10.10 Characteristics of common logic families

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>74</th>
<th>74LS</th>
<th>74HC</th>
<th>40BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum supply voltage (V)</td>
<td>5.25</td>
<td>5.25</td>
<td>5.5</td>
<td>18</td>
</tr>
<tr>
<td>Minimum supply voltage (V)</td>
<td>4.75</td>
<td>4.75</td>
<td>4.5</td>
<td>3</td>
</tr>
<tr>
<td>Static power dissipation (mW per gate at 100 kHz)</td>
<td>10</td>
<td>2</td>
<td>Negligible</td>
<td>Negligible</td>
</tr>
<tr>
<td>Dynamic power dissipation (mW per gate at 100 kHz)</td>
<td>10</td>
<td>2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Typical propagation delay (ns)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>105</td>
</tr>
<tr>
<td>Maximum clock frequency (MHz)</td>
<td>35</td>
<td>40</td>
<td>40</td>
<td>12</td>
</tr>
<tr>
<td>Speed-power product (pJ at 100 kHz)</td>
<td>100</td>
<td>20</td>
<td>1.2</td>
<td>11</td>
</tr>
<tr>
<td>Minimum output current (mA at $V_o = 0.4$ V)</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>1.6</td>
</tr>
<tr>
<td>Fan-out (number of standard loads that can be driven)</td>
<td>40</td>
<td>20</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Maximum input current (mA at $V_i = 0.4$ V)</td>
<td>−1.6</td>
<td>−0.4</td>
<td>0.001</td>
<td>−0.001</td>
</tr>
</tbody>
</table>
Procedure

Connect the circuit shown in Fig. 10.33 (see also Fig. 10.34 for the corresponding breadboard layout). Note that the green LED should become illuminated when the bistable is in the SET condition (i.e., when \( Q \) is at logic 1) and the red LED should become illuminated when the bistable is in the RESET condition. Note also that the 47 k\( \Omega \) resistors act as pull-up resistors. They are used to ensure that the respective input goes to logic 1 when the corresponding link is removed.

With both links in place (i.e., \( \text{SET} = 0 \) and \( \text{RESET} = 0 \)) observe and record (using a truth table) the state of the outputs.

Remove the RESET link (to make \( \text{RESET} = 1 \)) while leaving the SET link in place (to keep \( \text{SET} = 0 \)). Once again, observe and record the state of the outputs. Replace the RESET link (to make \( \text{RESET} = 0 \)) and check that the bistable does not change state. Now remove the SET link (to make \( \text{SET} = 1 \)). Once again, observe and record the state of the outputs. Replace the SET link (to make \( \text{SET} = 0 \)) and once again check that the bistable does not change state. Finally, remove both links (to make \( \text{SET} = 1 \) and \( \text{RESET} = 1 \)) and observe the state of the outputs in this disallowed state.

Conclusion

Comment on the truth table produced. Is this what you would expect? What happened when both SET and RESET inputs were at logic 1? Suggest a typical application for the circuit.

![Bistable circuit used in the Practical investigation. The LEDs are used to indicate the state of the outputs](image1)

**Figure 10.33**

![Bistable symbols](image2)

**Figure 10.35**
10.2 Show how a four-input OR gate can be made from three two-input OR gates.

10.3 Construct the truth table for the logic gate arrangement shown in Fig. 10.37.

10.4 Using only two-input NAND gates, show how each of the following logical functions can be satisfied:
   (a) two-input AND;
   (b) two-input OR;
   (c) four-input AND.
   In each case, use the minimum number of gates. (Hint: a two-input NAND gate can be made into an inverter by connecting its two inputs together.)

10.5 The rocket motor of an air-launched missile will operate if, and only if, the following conditions are satisfied:
   (i) ‘launch’ signal is at logic 1;
   (ii) ‘unsafe height’ signal is at logic 0;
   (iii) ‘target lock’ signal is at logic 1.
   Devise a suitable logic arrangement that will satisfy this requirement. Simplify your answer using the minimum number of logic gates.

10.6 An automatic sheet metal guillotine will operate if the following conditions are satisfied:
   (i) ‘guard lowered’ signal is at logic 1;
   (ii) ‘feed jam’ signal is at logic 0;
   (iii) ‘manual start’ signal is at logic 1.
   The sheet metal guillotine will also operate if the following conditions are satisfied:
   (i) ‘manual start’ signal is at logic 1;
   (ii) ‘test key’ signal is at logic 1.
   Devise a suitable logic arrangement that will satisfy this requirement. Use the minimum number of logic gates.

10.7 Devise a logic arrangement using no more than four two-input gates that will satisfy the truth table shown in Fig. 10.38.

10.8 Devise a logic arrangement that will produce the output waveform from the three input waveforms shown in Fig. 10.39.
10 Logic circuits

Label the inputs and outputs on your diagram.

10.12 Sketch the symbol of each of the following types of bistable:
(a) an R-S bistable;
(b) a D-type bistable;
(c) a J-K bistable.
Label your drawings clearly.

10.13 With the aid of a diagram, explain how a three-stage binary counter can be built using J-K bistables.

10.14 With the aid of a diagram, explain how a three-stage shift register can be built using J-K bistables.

10.15 Identify each of the logic devices shown in Fig. 10.40.

10.16 Explain, in relation to the scale of integration, what is meant by the terms (a) MSI and (b) VLSI.

Figure 10.37 See Questions 10.3 and 10.22

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 10.38 See Question 10.7

Figure 10.39 See Question 10.8.

10.9 A logic device is marked ‘74LS90 2798’. To which family and sub-family of logic does it belong and when was the device manufactured?

10.10 A logic family recognizes a logic 1 input as being associated with any voltage between 2.0 V and 5.5 V. The same family produces an output in the range 2.6 V to 5.0 V, corresponding to a logic 1 output. Determine the noise margin.

10.11 Sketch the circuit of a bistable using:
(a) two NAND gates;
(b) two NOR gates.

Figure 10.40 See Question 10.15
10 Logic circuits

10.17 Fig. 10.41 shows the internal schematic for a logic device. Identify the logic family to which this device belongs and state its logic function.

10.18 Specify typical logic levels for the logic device shown in Fig. 10.41. In relation to your answer, explain the significance of the indeterminate region and explain how this effects the noise margin of the device.

10.19 Sketch the logic gate arrangement of a four-input majority vote circuit. Using a truth table, briefly explain the operation of the circuit.

10.20 Show, with the aid of a logic diagram, how an exclusive-OR gate can be built using only two-input NAND gates.

10.21 Specify typical values for the power dissipation, propagation delay and maximum clock speed for (a) low-power Schottky TTL and (b) buffered CMOS.

10.22 Devise a logic gate arrangement using only two-input NAND gates that will perform the same logic function as the arrangement shown in Fig. 10.37. Simplify your answer as far as possible using the minimum number of logic gates.

Answers to these problems appear on page 417.
Chapter summary

Many of today’s complex electronic systems are based on the use of a microprocessor or microcontroller. Such systems comprise hardware that is controlled by software. If it is necessary to change the way that the system behaves it is the software (rather than the hardware) that is changed.

In this chapter we provide an introduction to microprocessors and explain, in simple terms, both how they operate and how they are used. We shall start by explaining some of the terminology that is used to describe different types of system that involve the use of a microprocessor or a similar device.
Microprocessor systems
Microprocessor systems are usually assembled on a single PCB comprising a microprocessor CPU together with a number of specialized support chips. These very large-scale integrated (VLSI) devices provide input and output to the system, control and timing, as well as storage for programs and data.

Typical applications for microprocessor systems include the control of complex industrial processes. Typical examples are based on families of chips such as the Z80CPU plus Z80PIO, Z80CTC, and Z80SIO.

Single-chip microcomputers
A single-chip microcomputer is a complete computer system (comprising CPU, RAM and ROM, etc.) in a single VLSI package. A single-chip microcomputer requires very little external circuitry in order to provide all of the functions associated with a complete computer system (but usually with limited input and output capability).

Single-chip microcomputers may be programmed using in-built programmable memories or via external memory chips. Typical applications of single-chip microcomputers include computer printers, instrument controllers, and displays. A typical example is the Z84C.

Microcontrollers
A microcontroller is a single-chip microcomputer that is designed specifically for control rather than general-purpose applications. They are often used to satisfy a particular control requirement, such as controlling a motor drive. Single-chip microcomputers, on the other hand, usually perform a variety of different functions and may control several processes at the same time.

Typical applications include control of peripheral devices such as motors, drives, printers, and minor sub-system components. Typical examples are the Z86E, 8051, 68705 and 89C51.

PIC microcontrollers
A PIC microcontroller is a general-purpose microcontroller device that is normally used in a stand-alone application to perform simple logic, timing and input/output control. PIC devices provide a flexible low-cost solution that very effectively bridges the gap between single-chip computers and the use of discrete logic and timer chips, as explained in Chapter 17.

A number of PIC and microcontroller devices have been produced that incorporate a high-level language interpreter. The resident interpreter allows developers to develop their programs’ languages such as BASIC rather than having to resort to more complex assembly language. This feature makes PIC microcontrollers very easy to use. PIC microcontrollers are used in ‘self-contained’ applications involving logic, timing and simple analogue-to-digital and digital-to-analogue conversion. Typical examples are the PIC12C508 and PIC16C620.

Programmed logic devices
While not an example of a microprocessor device, a programmed logic device (PLD) is a programmable chip that can carry out complex logical operations. For completeness, we have included a reference to such devices here.

PLDs are capable of replacing a large number of conventional logic gates, thus minimizing chip-count and reducing PCB sizes. Programming is relatively straightforward and simply requires the derivation of complex logic functions using Boolean algebra (see Chapter 10) or truth tables. Typical examples are the 16L8 and 22V10.

Programmable logic controllers
Programmable logic controllers (PLCs) are microprocessor-based systems that are used for controlling a wide variety of automatic processes, from operating an airport baggage handling system to brewing a pint of your favourite lager. PLCs are rugged and modular and they are designed specifically for operation in the process control environment.
The control program for a PLC is usually stored in one or more semiconductor memory devices. The program can be entered (or modified) by means of a simple hand-held programmer, a laptop controller, or downloaded over a local area network (LAN). PLC manufacturers include Allen Bradley, Siemens and Mitsubishi.

Microprocessor systems

The basic components of any microprocessor system (see Fig. 11.1) are:

(a) a central processing unit (CPU);
(b) a memory, comprising both ‘read/write’ and ‘read only’ devices (commonly called RAM and ROM, respectively);
(c) a means of providing input and output (I/O).

For example, a keypad for input and a display for output.

In a microprocessor system the functions of the CPU are provided by a single VLSI microprocessor chip (see Fig. 11.2). This chip is equivalent to many thousands of individual transistors. Semiconductor devices are also used to provide the read/write and read-only memory. Strictly speaking, both types of memory permit ‘random access’ since any item of data can be retrieved with equal ease regardless of its actual location within the memory. Despite this, the term ‘RAM’ has become synonymous with semiconductor read/write memory.

The basic components of the system (CPU, RAM, ROM and I/O) are linked together using a multiple-wire connecting system know as a bus (see Fig. 11.1). Three different buses are present; these are:

(a) the address bus used to specify memory locations;
(b) the data bus on which data are transferred between devices; and
(c) the control bus which provides timing and control signals throughout the system.

The number of individual lines present within the address bus and data bus depends upon the particular microprocessor employed. Signals on all lines, no matter whether they are used for address, data or control, can exist in only two basic states: logic 0 (low) or logic 1 (high). Data and addresses are represented by binary numbers (a sequence of 1s and 0s) that appear respectively on the data and address bus.
11 Microprocessors

Many microprocessors designed for control and instrumentation applications make use of an 8-bit data bus and a 16-bit address bus. Others have data and address buses which can operate with as many as 128-bits at a time.

The largest binary number that can appear on an 8-bit data bus corresponds to the condition when all eight lines are at logic 1. Therefore the largest value of data that can be present on the bus at any instant of time is equivalent to the binary number 11111111 (or 255). Similarly, the highest address that can appear on a 16-bit address bus is 1111111111111111 (or 65,535). The full range of data values and addresses for a simple microprocessor of this type is thus:

Data from 00000000 to 11111111
Addresses from 0000000000000000 to 1111111111111111

Data representation

Binary numbers – particularly large ones – are not very convenient. To make numbers easier to handle we often convert binary numbers to hexadecimal (base 16). This format is easier for mere humans to comprehend and offers the advantage over denary (base 10) in that it can be converted to and from binary with ease. The first 16 numbers in binary, denary and hexadecimal are shown in Table 11.1. A single hexadecimal character (in the range zero to F) is used to represent a group of four binary digits (bits). This group of four bits (or single hex character) is sometimes called a nibble.

A byte of data comprises a group of eight bits. Thus a byte can be represented by just two hexadecimal (hex) characters. A group of 16 bits (a word) can be represented by four hex characters, 32 bits (a double word by eight hex characters, and so on).

The value of a byte expressed in binary can be easily converted to hex by arranging the bits in groups of four and converting each nibble into hexadecimal using Table 11.1.

Note that, to avoid confusion about whether a number is hexadecimal or decimal, we often place a $ symbol before a hexadecimal number or add an H to the end of the number. For example, 64 means decimal ‘sixty-four’; whereas $64 means hexadecimal ‘six-four’, which is equivalent to decimal 100. Similarly, 7FH means hexadecimal ‘seven-F’, which is equivalent to decimal 127.

Example 11.1

Convert hexadecimal A3 into binary.

Solution

From Table 11.1, A = 1010 and 3 = 0011. Thus A3 in hexadecimal is equivalent to 10100011 in binary.

Example 11.2

Convert binary 11101000 binary to hexadecimal.

Solution

From Table 11.1, 1110 = E and 1000 = 8. Thus 11101000 in binary is equivalent to E8 in hexadecimal.

Table 11.1 Binary, denary and hexadecimal

<table>
<thead>
<tr>
<th>Binary (base 2)</th>
<th>Denary (base 10)</th>
<th>Hexadecimal (base 16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0010</td>
<td>2</td>
<td>2</td>
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<td>0011</td>
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<td>0100</td>
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<td>0110</td>
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<td>1000</td>
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<td>C</td>
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<td>1101</td>
<td>13</td>
<td>D</td>
</tr>
<tr>
<td>1110</td>
<td>14</td>
<td>E</td>
</tr>
<tr>
<td>1111</td>
<td>15</td>
<td>F</td>
</tr>
</tbody>
</table>
Data types

A byte of data can be stored at each address within the total memory space of a microprocessor system. Hence one byte can be stored at each of the 65,536 memory locations within a microprocessor system having a 16-bit address bus.

Individual bits within a byte are numbered from 0 (least significant bit) to 7 (most significant bit). In the case of 16-bit words, the bits are numbered from 0 (least significant bit) to 15 (most significant bit).

Negative (or signed) numbers can be represented using two’s complement notation where the leading (most significant) bit indicates the sign of the number (1 = negative, 0 = positive). For example, the signed 8-bit number 10000001 represents the denary number \(-1\).

The range of integer data values that can be represented as bytes, words and long words are shown in Table 11.2.

Data storage

The semiconductor ROM within a microprocessor system provides storage for the program code as well as any permanent data that requires storage. All of these data are referred to as non-volatile because they remain intact when the power supply is disconnected.

The semiconductor RAM within a microprocessor system provides storage for the transient data and variables that are used by programs. Part of the RAM is also used by the microprocessor as a temporary store for data while carrying out its normal processing tasks.

It is important to note that any program or data stored in RAM will be lost when the power supply is switched off or disconnected. The only exception to this is CMOS RAM, which is kept alive by means of a small battery. This battery-backed memory is used to retain important data, such as the time and date.

When expressing the amount of storage provided by a memory device we usually use Kilobytes (Kbyte). It is important to note that a Kilobyte of memory is actually 1,024 bytes (not 1,000 bytes). The reason for choosing the Kbyte rather than the kbyte (1,000 bytes) is that 1,024 happens to be the nearest power of 2 (note that \(2^{10} = 1,024\)).

The capacity of a semiconductor ROM is usually specified in terms of an address range and the number of bits stored at each address. For example, 2 K \(\times\) 8 bits (capacity 2 Kbytes), 4 K \(\times\) 8 bits (capacity 4 Kbytes), and so on. Note that it is not always necessary (or desirable) for the entire memory space of a microprocessor to be populated by memory devices.

The microprocessor

The microprocessor central processing unit (CPU) forms the heart of any microprocessor or microcomputer system computer and, consequently, its operation is crucial to the entire system.

The primary function of the microprocessor is that of fetching, decoding and executing instructions resident in memory. As such, it must be able to transfer data from external memory into its own internal registers and vice versa. Furthermore, it must operate predictably, distinguishing, for example, between an operation contained within an instruction and any accompanying addresses of read/write memory locations. In addition, various system housekeeping tasks need to be performed including being able to suspend normal processing in order to respond to an external device that needs attention.

The main parts of a microprocessor CPU are:

(a) registers for temporary storage of addresses and data;
(b) an arithmetic logic unit (ALU) that performs arithmetic and logic operations;
(c) a unit that receives and decodes instructions; and
(d) a means of controlling and timing operations within the system.

<table>
<thead>
<tr>
<th>Table 11.2 Data types</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data type</strong></td>
</tr>
<tr>
<td>Unsigned byte</td>
</tr>
<tr>
<td>Signed byte</td>
</tr>
<tr>
<td>Unsigned word</td>
</tr>
<tr>
<td>Signed word</td>
</tr>
</tbody>
</table>
Fig. 11.3 shows the principal internal features of a typical 8-bit microprocessor. We will briefly explain each of these features in turn.

**Accumulator**

The accumulator functions as a source and destination register for many of the basic microprocessor operations. As a *source register* it contains the data that will be used in a particular operation, while as a *destination register* it will be used to hold the result of a particular operation. The accumulator (or *A-register*) features in a very large number of microprocessor operations, consequently more reference is made to this register than any others.

**Instruction register**

The instruction register provides a temporary storage location in which the current microprocessor instruction is held while it is being decoded. Program instructions are passed into the microprocessor, one at time, through the data bus.

On the first part of each *machine cycle*, the instruction is fetched and decoded. The instruction is executed on the second (and subsequent) machine cycles. Each machine cycle takes a finite time (usually less than a microsecond) depending upon the frequency of the microprocessor’s clock.

**Data bus (D0 to D7)**

The external data bus provides a highway for data that links all of the system components (such as random access memory, read-only memory and input/output devices) together. In an 8-bit system, the data bus has eight data lines, labelled D0 (the *least significant bit*) to D7 (*the most significant bit*) and data are moved around in groups of eight bits, or *bytes*. With a 16-bit data bus the data lines are labelled D0 to D15, and so on.
Program counter

Programs consist of a sequence of instructions that are executed by the microprocessor. These instructions are stored in external random access memory (RAM) or read-only memory (ROM). Instructions must be fetched and executed by the microprocessor in a strict sequence. By storing the address of the next instruction to be executed, the program counter allows the microprocessor to keep track of where it is within the program. The program counter is automatically incremented when each instruction is executed.

Address bus buffer

The address bus buffer is a temporary register through which addresses (in this case comprising 16 bits) pass on their way out of the microprocessor. In a simple microprocessor, the address buffer is unidirectional with addresses placed on the address bus during both read and write operations. The address bus lines are labelled A0 to A15, where A0 is the least significant address bus line and A15 is the most significant address bus line. Note that a 16-bit address bus can be used to communicate with 65,536 individual memory locations. At each location a single byte of data is stored.

Control bus

The control bus is a collection of signal lines that are both used to control the transfer of data around the system and also to interact with external devices. The control signals used by microprocessors tend to differ with different types: however the following are commonly found:

READ an output signal from the microprocessor that indicates that the current operation is a read operation.

WRITE an output signal from the microprocessor that indicates that the current operation is a write operation.

RESET a signal that resets the internal registers and initializes the program counter so that the program can be re-started from the beginning.

Data bus buffer

The data bus buffer is a temporary register through which bytes of data pass on their way into, and out of, the microprocessor. The buffer is thus referred to as bi-directional with data passing out of the microprocessor on a write operation and into the processor during a read operation. The direction of data transfer is determined by the control unit as it responds to each individual program instruction.

Internal data bus

The internal data bus is a high-speed data highway that links all of the microprocessor’s internal elements together. Data are constantly flowing backwards and forwards along the internal data bus lines.

General-purpose registers

Many microprocessor operations (for example, adding two 8-bit numbers together) require the use of more than one register. There is also a requirement for temporarily storing the partial result of an operation while other operations take place. Both of these needs can be met by providing a number of general-purpose registers. The use to which these registers are put is left mainly up to the programmer.

Stack pointer

When the time comes to suspend a particular task in order to briefly attend to something else, most microprocessors make use of a region of external random access memory (RAM) known as a stack. When the main program is interrupted, the microprocessor temporarily places in the stack the contents of its internal registers together with the address of the next instruction in the main program. When the interrupt has been attended to, the microprocessor recovers the data that have been stored temporarily in the stack together with the address of the next instruction within the main program. It is thus able to return to the main program exactly where it left off and with all the data preserved in its registers. The stack pointer is simply a register that contains the address of the last used stack location.
11 Microprocessors

IRQ interrupt request from an external device attempting to gain the attention of the microprocessor (the request may be obeyed or ignored according to the state of the microprocessor at the time that the interrupt request is received).

NMI non-maskable interrupt (i.e. an interrupt signal that cannot be ignored by the microprocessor).

Address bus (A0 to A15)
The address bus provides a highway for addresses that links with all of the system components (such as random access memory, read-only memory and input/output devices).

In a system with a 16-bit address bus, there are 16 address lines, labelled A0 (the least significant bit) to A15 (the most significant bit). In a system with a 32-bit address bus there are 32 address lines, labelled A0 to A31, and so on.

Instruction decoder
The instruction decoder is nothing more than an arrangement of logic gates that acts on the bits stored in the instruction register and determines which instruction is currently being referenced. The instruction decoder provides output signals for the microprocessor’s control unit.

Control unit
The control unit is responsible for organizing the orderly flow of data within the microprocessor as well as generating, and responding to, signals on the control bus. The control unit is also responsible for the timing of all data transfers. This process is synchronized using an internal or external clock signal (not shown in Fig. 11.3).

Arithmetic logic unit (ALU)
As its name suggests, the ALU performs arithmetic and logic operations. The ALU has two inputs (in this case these are both 8 bits wide). One of these inputs is derived from the accumulator while the other is taken from the internal data bus via a temporary register (not shown in Fig. 11.3). The operations provided by the ALU usually include addition, subtraction, logical AND, logical OR, logical exclusive-OR, shift left, shift right, etc. The result of most ALU operations appears in the accumulator.

Flag register (or status register)
The result of an ALU operation is sometimes important in determining what subsequent action takes place. The flag register contains a number of individual bits that are set or reset according to the outcome of an ALU operation. These bits are referred to as flags. The following flags are available in most microprocessors:

ZERO the zero flag is set when the result of an ALU operation is zero (i.e. a byte value of 00000000).

CARRY the carry flag is set whenever the result of an ALU operation (such as addition) generates a carry bit (in other words, when the result cannot be contained within an 8-bit register).

INTERRUPT the interrupt flag indicates whether external interrupts are currently enabled or disabled.

Clocks
The clock used in a microprocessor system is simply an accurate and stable square wave generator. In most cases the frequency of the square wave generator is determined by a quartz crystal. A simple 4 MHz square wave clock oscillator (together with the clock waveform that is produced) is shown in Fig. 11.4. Note that one complete clock cycle is sometimes referred to as a T-state.

Microprocessors sometimes have an internal clock circuit, in which case the quartz crystal (or other resonant device) is connected directly to pins on the microprocessor chip. In Fig. 11.5(a) an external clock is shown connected to a microprocessor, while in Fig.11.5(b) an internal clock oscillator is used.

Microprocessor operation
The majority of operations performed by a microprocessor involve the movement of data. Indeed, the program code (a set of instructions stored in ROM or RAM) must itself
Each cycle of CPU operation is known as a machine cycle. Program instructions may require several machine cycles (typically between two and five). The first machine cycle in any cycle consists of an instruction fetch (the instruction code is read from the memory) and it is known as the M1 cycle. Subsequent cycles M2, M3 and so on depend on the type of instruction that is being executed. This fetch–execute sequence is shown in Fig. 11.7.

Microprocessors determine the source of data (when it is being read) and the destination of data (when it is being written) by placing a unique address on the address bus. The address at which the data are to be placed (during a write operation) or from which they are to be fetched (during a read operation) can either constitute part of the memory of the system (in which case it may be within ROM or RAM) or it can be considered to be associated with input/output (I/O).

Since the data bus is connected to a number of VLSI devices, an essential requirement of such chips (e.g. ROM or RAM) is that their data outputs should be capable of being isolated from the bus be fetched from memory prior to execution. The microprocessor thus performs a continuous sequence of instruction fetch and execute cycles. The act of fetching an instruction code (or operand or data value) from memory involves a read operation while the act of moving data from the microprocessor to a memory location involves a write operation – see Fig. 11.6.
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whenever necessary. These chips are fitted with select or enable inputs that are driven by address decoding logic (not shown in Fig. 11.7). This logic ensures that ROM, RAM and I/O devices never simultaneously attempt to place data on the bus! The inputs of the address decoding logic are derived from one, or more, of the address bus lines. The address decoder effectively divides the available memory into blocks corresponding to a particular function (ROM, RAM, I/O, etc). Hence, where the processor is reading and writing to RAM, for example, the address decoding logic will ensure that only the RAM is selected while the ROM and I/O remain isolated from the data bus. Within the CPU, data are stored in several registers. Registers themselves can be thought of as a simple pigeon-hole arrangement that can store as many bits as there are holes available. Generally, these devices can store groups of 16 or 32 bits. Additionally, some registers may be configured as either one register of 16 bits or two registers of 32 bits.

Some microprocessor registers are accessible to the programmer, whereas others are used by the microprocessor itself. Registers may be classified as either general purpose or dedicated. In the latter case a particular function is associated with the register, such as holding the result of an operation or signalling the result of a comparison. A typical microprocessor and its register model is shown in Fig. 11.8.
The arithmetic logic unit
The ALU can perform arithmetic operations (addition and subtraction) and logic (complementation, logical AND, logical OR, etc). The ALU operates on two inputs (16 or 32 bits in length depending upon the CPU type) and it provides one output (again of 16 or 32 bits). In addition, the ALU status is preserved in the flag register so that, for example, an overflow, zero or negative result can be detected.

The control unit is responsible for the movement of data within the CPU and the management of control signals, both internal and external. The control unit asserts the requisite signals to read or write data as appropriate to the current instruction.

Input and output
The transfer of data within a microprocessor system involves moving groups of 8, 16 or 32 bits using the bus architecture described earlier. Consequently it is a relatively simple matter to transfer data into and out of the system in parallel form. This process is further simplified by using a Programmable Parallel I/O device (a Z80PIO, 8255 or equivalent VLSI chip). This device provides registers for the temporary storage of data that not only buffer the data but also provide a degree of electrical isolation from the system data bus.

Parallel data transfer is primarily suited to high-speed operation over relatively short distances, a typical example being the linking of a microcomputer to an adjacent printer. There are, however, some applications in which parallel data transfer is inappropriate, the most common example being data communication by means of telephone lines. In such cases data must be sent serially (one bit after another) rather than in parallel form.

To transmit data in serial form, the parallel data from the microprocessor must be reorganized into a stream of bits. This task is greatly simplified by using an LSI interface device that contains a shift register that is loaded with parallel data from the data bus. This data are then read out as a serial bit stream by successive shifting. The reverse process, serial-to-parallel conversion, also uses a shift register. Here data are loaded in serial form, each bit shifting further into the register until it becomes full. Data are then placed simultaneously on the parallel output lines. The basic principles of parallel-to-serial and serial-to-parallel data conversion are illustrated in Fig. 11.9.

An example program
The following example program (see Table 11.3) is written in assembly code. The program transfers 8-bit data from an input port (Port A), complements (i.e. inverts) the data (by changing 0s to 1s and 1s to 0s in every bit position) and then outputs the result to an output port (Port B). The program repeats indefinitely.

Just three microprocessor instructions are required to carry out this task together with a fourth (jump) instruction that causes the three instructions to be repeated over and over again. A program of this sort is most easily written in assembly code which consists of a series of easy
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Table 11.3 A simple example program

<table>
<thead>
<tr>
<th>Address</th>
<th>Data</th>
<th>Assembly code</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>DB FF</td>
<td>IN A, (FFH)</td>
<td>Get a byte from Port A</td>
</tr>
<tr>
<td>2002</td>
<td>2F</td>
<td>CPL</td>
<td>Invert the byte</td>
</tr>
<tr>
<td>2003</td>
<td>D3 FE</td>
<td>OUT (FEH), A</td>
<td>Output the byte to Port B</td>
</tr>
<tr>
<td>2005</td>
<td>C3 00 20</td>
<td>JP 2000</td>
<td>Go round again</td>
</tr>
</tbody>
</table>

Figure 11.10 (a) Flowchart for the example program and (b) the eight bytes of program code stored in memory

A microcontroller system

Fig. 11.11 shows the arrangement of a typical microcontroller system. The sensed quantities (temperature, position, etc.) are converted to corresponding electrical signals by means of a number of sensors. The outputs from the sensors (in either digital or analogue form) are passed as input signals to the microcontroller. The microcontroller also accepts inputs from the user. These user-set options typically include target values for variables (such as desired room temperature), limit values (such as maximum shaft speed) or time constraints (such as ‘on’ time and ‘off’ time, delay time, etc.).

The operation of the microcontroller is controlled by a sequence of software instructions known as a control program. The control program operates continuously, examining inputs from sensors, user settings and time data before making changes to the output signals sent to one or more controlled devices.

The controlled quantities are produced by the controlled devices in response to output signals from the microcontroller. The controlled device generally converts energy from one form into energy in another form. For example, the controlled device might be an electrical heater that converts electrical energy from the AC mains supply into heat energy, thus producing a given temperature (the controlled quantity).
In most real-world systems there is a requirement for the system to be automatic or self-regulating. Once set, such systems will continue to operate without continuous operator intervention. The output of a self-regulating system is fed back to its input in order to produce what is known as a closed-loop system. A good example of a closed-loop system is a heating control system that is designed to maintain a constant room temperature and humidity within a building regardless of changes in the outside conditions.

In simple terms, a microcontroller must produce a specific state on each of the lines connected to its output ports in response to a particular combination of states present on each of the lines connected to its input ports (see Fig. 11.11). Microcontrollers must also have a central processing unit (CPU) capable of performing simple arithmetic, logical and timing operations.

The input port signals can be derived from a number of sources, including:
- switches (including momentary action push-buttons);
- sensors (producing logic-level compatible outputs);
- keypads (both encoded and unencoded types).

The output port signals can be connected to a number of devices, including:
- LED indicators (both individual and multiple bar types);
- LED seven-segment displays (via a suitable interface);
- motors and actuators (both linear and rotary types) via a suitable buffer/driver or a dedicated interface;
- relays (both conventional electromagnetic types and optically couple solid-state types);
- transistor drivers and other solid-state switching devices.

**Input devices**

Input devices supply information to the computer system from the outside world. In an ordinary personal computer, the most obvious input device is the keyboard. Other input devices available on a PC are the mouse (pointing device), scanner, and modem. Microcontrollers use much simpler input devices. These need be nothing more than individual switches or contacts that make and break, but many other types of device are also used including many types of sensor that provide logic-level outputs (such as float switches, proximity detectors, light sensors, etc.).

![Figure 11.11 A microcontroller system with typical inputs and outputs](image-url)
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It is important to note that, in order to be connected directly to the input port of a microcontroller, an input device must provide a logic compatible signal. This is because microcontroller inputs can only accept digital input signals with the same voltage levels as the logic power source. The 0 V ground level (often referred to as Vss in the case of a CMOS microcontroller) and the positive supply (Vdd in the case of a CMOS microcontroller) is invariably 5 V ± 5%. A level of approximately 0V indicates a logic 0 signal and a voltage approximately equal to the positive power supply indicates a logic 1 signal. Other input devices may sense analogue quantities (such as velocity) but use a digital code to represent their value as an input to the microcontroller system. Some microcontrollers provide an internal analogue-to-digital converter (ADC) in order to greatly simplify the connection of analogue sensors as input devices, but where this facility isn’t available it will be necessary to use an external ADC which usually takes the form of a single integrated circuit. The resolution of the ADC will depend upon the number of bits used and 8-, 10- and 12-bit devices are common in control applications.

Output devices

Output devices are used to communicate information or actions from a computer system to the outside world. In a personal computer system, the most common output device is the flat-screen display. Other output devices include printers and modems. As with input devices, microcontroller systems often use much simpler output devices. These may be nothing more than LEDs, piezoelectric sounders, relays and motors.

In order to be connected directly to the output port of a microcontroller, an output device must, once again, be able to accept a logic compatible signal. Where analogue quantities (rather than simple digital on/off operation) are required at the output a digital-to-analogue converter (DAC) will be needed. All of the functions associated with a DAC can be provided by a single integrated circuit. As with an ADC, the output resolution of a DAC depends on the number of bits and 8, 10 and 12 bits are common in control applications.

Interface circuits

Finally, where input and output signals are not logic compatible (i.e. when they are outside the range of signals that can be connected directly to the microcontroller) some additional interface circuitry may be required in order to shift the voltage levels or to provide additional current drive. Additional circuitry may also be required when a load (such as a relay or motor) requires more current than is available from a standard logic device or output port. For example, a common range of interface circuits (solid-state relays) is available that will allow a microcontroller to be easily interfaced to an a.c. mains-connected load. It then becomes possible for a small microcontroller (operating from only a 5 V d.c. supply) to control a central heating system operating from 240 V a.c. mains.

Figure 11.12 An analogue input signal can be connected to a microcontroller input port via an analogue-to-digital converter (ADC)
Practical investigation

Objective

To investigate the operation of a Z80 microprocessor using a simulator to run a simple assembly language program.

Simulator

Oshonsoft Z80 Simulator or similar Integrated Development Environment (IDE). The Oshonsoft IDE can be downloaded from www.oshonsoft.com. Alternatively, a Z80 development system with resident assembler and I/O ports can be used (note that port addresses used in the Practical Investigation may need to be modified in order to agree with those available).

System configuration and required operation

For the purposes of this investigation we shall assume that the system has eight input switches connected to port address 80H and eight LED indicators connected to output port address 82H. The LEDs are to be operated from their corresponding input switches on the following basis:

**Inputs (Port 80H)**

- Switch ‘on’ = logic 1
- Switch ‘off’ = logic 0

**Outputs (Port 82H)**

- Logic 1 = LED ‘off’
- Logic 0 = LED ‘on’

Note that in order to illuminate an LED the logic 1 input must be inverted in order to produce a logic 0 output. Hence it will be necessary to read the byte from port 80H, invert (i.e. complement) it, and then output it to port 82H.

The program is to continue to run, detecting the state of the input switches and turning the appropriate LEDs on, until such a time as all of the input switches are set to the ‘off’ position. If this situation is detected the program is to halt.

Procedure

Start the Z80 Simulator (see Fig. 11.14) then select Tools and Assembler and enter the assembly language code shown in Fig. 11.15. When complete select File and Save your assembly language source code with a suitable name (e.g. switch.asm).

Next select Tools and Assemble. Correct any errors in the assembly language source code and repeat the process until the source code assembles without error. Save the final assembly code and then quit the assembler in order to return to the main IDE screen.

Select File and Load Program and then select the object code (switch.obj) that has just been produced by the assembler. You will then need
11 Microprocessors

Figure 11.14 The Z80 Simulator Integrated Development Environment (IDE)

Figure 11.15 Assembly language source code for the Practical investigation
to configure the input and output ports before you run the program. You can do this by selecting Tools and Peripheral Devices (see Fig. 11.16). Configure the peripheral devices so that the input port address is 80H and the output port address is 82H. Then set some of the input bits by clicking on the indicators (which will change colour).

Next, return to the main IDE, select Tools and Simulation Log Viewer and then Rate and Slow. Finally, select Simulation and Start (or press the F1 key). You will see the program instructions and the contents of the Z80’s registers displayed as the program is executed (see Fig. 11.17). If you have left the Peripheral Devices window open you will also be able to change the switch settings and observe the effect that this has. Finally, you should set all of the switches to the ‘off’ position and check that the program exits from the loop and reaches the HALT instruction.

A further program

If you have been able to run the simple program successfully you might like to try your hand at developing your own assembly language program using some of the assembly language instructions shown in Table 11.4. This program should shift the bits on output Port 82H to the left by the number of places indicated by data from Port 80H. The two ports can be preset before running the program.
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Table 11.4 Some selected Z80 assembly language instructions

<table>
<thead>
<tr>
<th>Assembly code</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD A, data</td>
<td>Load the Accumulator with 8-bit data</td>
</tr>
<tr>
<td>LD B, data</td>
<td>Load the B register with 8-bit data</td>
</tr>
<tr>
<td>LD C, data</td>
<td>Load the C register with 8-bit data</td>
</tr>
<tr>
<td>LD C,A</td>
<td>Load the C register with 8-bit data from the Accumulator</td>
</tr>
<tr>
<td>CP B</td>
<td>Compare the value in the B register with the value in the Accumulator</td>
</tr>
<tr>
<td>DEC C</td>
<td>Decrement the C register</td>
</tr>
<tr>
<td>CPL</td>
<td>Complement (i.e. invert) the contents of the Accumulator</td>
</tr>
<tr>
<td>SLA A</td>
<td>Shift the contents of the Accumulator left by one bit</td>
</tr>
<tr>
<td>JP NZ, label</td>
<td>Jump to the symbolic address label of the Zero flag has been set</td>
</tr>
<tr>
<td>HALT</td>
<td>Halt (suspend program execution)</td>
</tr>
<tr>
<td>IN A, (port)</td>
<td>Input the data from the specified port to the Accumulator</td>
</tr>
<tr>
<td>OUT (port), A</td>
<td>Output the data from the Accumulator to the specified port</td>
</tr>
</tbody>
</table>

11.5 How many unique addresses are available to a microprocessor CPU that has a 20-bit address bus?

11.6 What is the largest unsigned data value that can appear on a 10-bit data bus?

11.7 What is the largest negative data value that can be represented using signed 16-bit binary numbers?

11.8 The following fragment of assembly language code is executed using a Z80 microprocessor:

IN A, (FEH)
CPL
OUT (FFH), A
HALT

(a) What are the addresses of the input and output ports?
(b) If a data value of 10101111 appears at the input port what value will appear at the output port after the code has been executed?

11.9 Give examples of (a) two input devices and (b) two output devices commonly used in microprocessor systems.

Symbols introduced in this chapter

Figure 11.18 Symbols introduced in this chapter

Problems

11.1 Convert 3A hexadecimal to binary.
11.2 Convert 11000010 binary to hexadecimal.
11.3 Convert 63 decimal to
   (a) binary
   (b) hexadecimal.
11.4 Which of the following numbers is the largest?
   (a) 19H
   (b) $13$
   (c) 3310
   (d) 111012.

Figure 11.19 See Questions 11.10 and 11.11

Figure 11.20 See Question 11.12
11.10 Explain the purpose of the circuit shown in Fig. 11.19 and state the function of the components marked A, B and C.

11.11 Sketch two cycles of the typical output waveform produced by the circuit shown in Fig. 11.19. Include labelled axes of time and voltage.

11.12 Identify, and briefly explain the purpose of, the features labelled P, Q, R, S, T, U and V in the microcomputer system shown in Fig. 11.20.

11.13 Explain the function of four common control bus signals used in a microcomputer system.

11.14 Explain the need for an ADC when a temperature sensor is to be interfaced to a microcomputer system.

Answers to these problems appear on page 417.
The 555 timer

Chapter summary
The 555 timer is without doubt one of the most versatile integrated circuit chips ever produced. Not only is it a neat mixture of analogue and digital circuitry, but its applications are virtually limitless in the world of timing and digital pulse generation. The device also makes an excellent case study for newcomers to electronics because it combines a number of important concepts and techniques.
Internal features

To begin to understand how timer circuits operate, it is worth spending a few moments studying the internal circuitry of the 555 timer (see Fig 12.1). Essentially, the device comprises two operational amplifiers (used as comparators – see page 169) together with an R–S bistable element (see page 192). In addition, an inverting buffer (see page 190) is incorporated so that an appreciable current can be delivered to a load. The main features of the device are shown in Table 12.1.

Unlike standard TTL logic devices, the 555 timer can both source and sink current. It’s worth taking a little time to explain what we mean by these two terms:

(a) When sourcing current, the 555’s output (pin 3) is in the high state and current will then flow out of the output pin into the load and down to 0 V, as shown in Fig. 12.2(a).

(b) When sinking current, the 555’s output (pin 3) is in the low state in which case current will flow from the positive supply (V_{CC}) through the load and into the output (pin 3), as shown in Fig. 12.2(b).

Returning to Fig. 12.1, the single transistor switch, TR1, is provided as a means of rapidly discharging an external timing capacitor. Because the series chain of resistors, R1, R2 and R3, all have identical values, the supply voltage (V_{CC}) is divided equally across the three resistors. Hence the voltage at the non-inverting input of IC1 is one-third of the supply voltage (V_{CC}) while that at the inverting input of IC2 is two-thirds of the supply voltage (V_{CC}). Hence if V_{CC} is 9 V, 3 V will appear at each resistor and the upper comparator will have 6 V applied to its inverting input while the lower comparator will have 3 V at its non-inverting input.

The 555 family

The standard 555 timer is housed in an eight-pin dual-in-line (DIL) package and operates from supply rail voltages of between 4.5 V and 15 V. This, of course, encompasses the normal range for TTL devices (5 V ± 5%) and thus the device is ideally suited for use with TTL circuitry.

Several versions of the 555 timer are available, including low-power (CMOS) and dual versions, as follows.

Low-power (CMOS) 555

This device is a CMOS version of the 555 timer that is both pin and function compatible with its standard counterpart. By virtue of its CMOS technology the device operates over a somewhat wider range of supply voltages (2 V to 18 V) and consumes minimal operating current (120 mA typical for an 18 V supply). Note that, by virtue of the low-power CMOS technology employed, the device does not have the same output current drive as that possessed by its standard counterparts. It can, however, supply up to two standard TTL loads.

Dual 555 timer (e.g. NE556A)

This is a dual version of the standard 555 timer housed in a 14-pin DIL package. The two devices

<table>
<thead>
<tr>
<th>Feature</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A potential divider comprising R1, R2 and R3 connected in series. Since all three resistors have the same values the input voltage (V_{cc}) will be divided into thirds, i.e. one-third of V_{cc} will appear at the junction of R1 and R2 while two-thirds of V_{cc} will appear at the junction of R1 and R2.</td>
</tr>
<tr>
<td>B</td>
<td>Two operational amplifiers connected as comparators. The operational amplifiers are used to examine the voltages at the threshold and trigger inputs and compare these with the fixed voltages from the potential divider (two-thirds and one-third of V_{cc} respectively).</td>
</tr>
<tr>
<td>C</td>
<td>An R–S bistable stage. This stage can be either set or reset depending upon the output from the comparator stage. An external reset input is also provided.</td>
</tr>
<tr>
<td>D</td>
<td>An open-collector transistor switch. This stage is used to discharge an external capacitor by effectively shorting it out whenever the base of the transistor is driven positive.</td>
</tr>
<tr>
<td>E</td>
<td>An inverting power amplifier. This stage is capable of sourcing and sinking enough current (well over 100 mA in the case of a standard 555 device) to drive a small relay or another low-resistance load connected to the output.</td>
</tr>
</tbody>
</table>
12 The 555 timer

Figure 12.1 Internal arrangement of a 555 timer

Figure 12.2 Loads connected to the output of a 555 timer: (a) current sourced by the timer when the output is high (b) current sunk by the timer when the output is low

may be used entirely independently and share the same electrical characteristics as the standard 555.

Low-power (CMOS) dual 555 (e.g. ICM75561PA)

This is a dual version of the low-power CMOS 555 timer contained in a 14-pin DIL package. The two devices may again be used entirely independently and share the same electrical characteristics as the low-power CMOS 555.

Pin connecting details for the above devices can be found in Appendix 4.

Monostable pulse generator

Fig. 12.3 shows a standard 555 timer operating as a monostable pulse generator. The monostable timing period (i.e. the time for which the output is high) is initiated by a falling edge trigger pulse applied to the trigger input (pin 2) (see Fig. 12.4).

When this falling edge trigger pulse is received and falls below one-third of the supply voltage, the output of IC2 (Fig. 12.1) goes high and the bistable will be placed in the set state. The
The period of the 555 monostable output can be changed very easily by simply altering the values of the timing resistor, \( R \), and/or timing capacitor, \( C \). Doubling the value of \( R \) will double the timing period. Similarly, doubling the value of \( C \) will double the timing period.

Values for \( C \) and \( R \) can be selected over quite a wide range but it is worth noting that the performance of the timer may become unpredictable if the values of these components are outside the recommended range:

\[
C = 470 \ \text{pF} \text{ to } 470 \ \text{μF}
\]

\[
R = 1 \ \text{kΩ} \text{ to } 3.3 \ \text{MΩ}
\]

For any particular monostable timing period, the required values for \( C \) and \( R \) can be determined from the formula shown earlier or by using the graph shown in Fig. 12.5. The output period can be easily adjusted by making \( R \) a preset resistor with a value of about twice that of the calculated value.

**Example 12.1**

Design a timer circuit that will produce a 10 ms pulse when a negative-going trigger pulse is applied to it.

**Solution**

Using the circuit shown in Fig. 12.4, the value of monostable timing period can be calculated from the formula:

\[
t_{\text{on}} = 1.1 \ \frac{C}{R}
\]

We need to choose an appropriate value for \( C \) that is in the range stated earlier. Since we require a fairly modest time period we will choose a mid-range value for \( C \). This should help to ensure that the value of \( R \) is neither too small nor too large. A value of 100 nF should be appropriate and should also be easy to obtain. Making \( R \) the subject of the formula and substituting for \( C = 100 \ \text{nF} \):

\[
R = \frac{t_{\text{on}}}{1.1C} = \frac{10 \ \text{ms}}{1.1 \times 100 \ \text{nF}} = \frac{10 \times 10^{-3}}{110 \times 10^{-9}}
\]

From which:

\[
R = \frac{10}{110} \times 10^6 = 0.091 \times 10^6 \ \Omega = 91 \ \text{kΩ}
\]

Alternatively, the chart shown in Fig. 12.5 can be used.
12 The 555 timer

Example 12.2
Design a timer circuit that will produce a +5 V output for a period of 60 s when a 'start' button is operated. The time period is to be aborted when a 'stop' button is operated.

Solution
For the purposes of this question we shall assume that the 'start' and 'stop' buttons both have normally open (NO) actions.

The value of monostable timing period can be calculated from the formula:

\[ t_{on} = 1.1 \times C \times R \]

We need to choose an appropriate value for \( C \) that is in the range stated earlier. Since we require a fairly long time period we will choose a relatively large value of \( C \) in order to avoid making the value of \( R \) too high. A value of 100 \( \mu F \) should be appropriate and should also be easy to obtain. Making \( R \) the subject of the formula and substituting for \( C = 100 \mu F \) gives:

\[ R = \frac{t_{on}}{1.1C} = \frac{60 \text{ s}}{1.1 \times 100 \mu F} = \frac{60}{110 	imes 10^{-6}} \]

From which:

\[ R = \frac{60}{110} \times 10^6 = 0.545 \times 10^6 \Omega = 545 \text{ k}\Omega \]

In practice 560 k\( \Omega \) (the nearest preferred value – see page 23) would be adequate.

The 'start' button needs to be connected between pin 2 and ground while the 'stop' button needs to be connected between pin 4 and ground. Each of the inputs requires a pull-up resistor to ensure that the input is taken high when the switch is not being operated. The precise value of the 'pull-up' resistor is unimportant and a value of 10 k\( \Omega \) will be perfectly adequate in this application. The complete circuit of the 60 s timer is shown in Fig. 12.6.

Astable pulse generator
Fig. 12.7 shows how the standard 555 can be used as an astable pulse generator. In order to understand how this circuit operates, assume that

![Figure 12.5 Chart for determining values of \( C \), \( t_{on} \) and \( R \) for a 555 operating in monostable mode. The dotted line shows how a 10 ms pulse will be produced when \( C = 100 \text{ nF} \) and \( R = 91 \text{ k}\Omega \) (see Example 12.1)]

![Figure 12.6 60 s timer (see Example 12.2)]
the output (pin 3) is initially high and that TR1 (Fig. 12.1) is in the non-conducting state. The capacitor, \( C \), will begin to charge with current supplied by means of series resistors, \( R_1 \) and \( R_2 \).

When the voltage at the threshold input (pin 6) exceeds two-thirds of the supply voltage the output of the upper comparator, IC1, will change state and the bistable will become reset due to voltage transition that appears at \( R \). This, in turn, will make the inverted Q output go high, turning TR1 at the same time. Due to the inverting action of the buffer, IC4, the final output (pin 3) will go low.

The capacitor, \( C \), will now discharge, with current flowing through \( R_2 \) into the collector of TR1. At a certain point, the voltage appearing at the trigger input (pin 2) will have fallen back to one-third of the supply voltage, at which point the lower comparator will change state and the voltage transition at \( S \) will return the bistable to its original set condition. The inverted Q output then goes low, TR1 switches off (no longer conducting), and the output (pin 3) goes high. Thereafter, the entire charge/discharge cycle is repeated indefinitely.

The output waveform produced by the circuit of Fig. 12.7 is shown in Fig. 12.8. The waveform has the following properties:

Time for which output is high:

\[
t_{\text{on}} = 0.693 \cdot \frac{C}{R_1 + R_2}
\]

Time for which output is low:

\[
t_{\text{off}} = 0.693 \cdot \frac{C}{R_2}
\]
from the formulae shown earlier. Alternatively, the graph shown in Fig. 12.9 can be used when $R_1$ and $R_2$ are equal in value (corresponding to a 67% duty cycle).

**Square wave generators**

Because the high time ($t_{on}$) is always greater than the low time ($t_{off}$), the mark to space ratio produced by a 555 timer can never be made equal to (or less than) unity. This could be a problem if we need to produce a precise square wave in which $t_{on} = t_{off}$. However, by making $R_2$ very much larger than $R_1$, the timer can be made to produce a reasonably symmetrical square wave output (note that the minimum recommended value for $R_1$ is 1 kΩ – see earlier).

If $R_2 \gg R_1$ the expressions for p.r.f. and duty cycle simplify to:

$$p.r.f. = \frac{0.72}{CR_2}$$

$$\frac{t_{on}}{t_{on} + t_{off}} = \frac{R_2}{2R_2} \times 100\% = \frac{1}{2} \times 100\% = 50\%$$

**Example 12.3**

Design a pulse generator that will produce a p.r.f. of 10 Hz with a 67% duty cycle.

**Solution**

Using the circuit shown in Fig. 12.7, the value of p.r.f. can be calculated from:

$$p.r.f. = \frac{1.44}{C(R_1 + 2R_2)}$$

Since the specified duty cycle is 67% we can make $R_1$ equal to $R_2$. Hence if $R = R_1 = R_2$ we obtain the following relationship:

$$p.r.f. = \frac{1.44}{C(R + 2R)} = \frac{1.44}{3CR} = \frac{0.48}{CR}$$

We need to choose an appropriate value for $C$ that is in the range stated earlier. Since we require a fairly low value of p.r.f. we will choose a value for $C$ of 1 μF. This should help to ensure that the value of $R$ is neither too small nor too large. A value of 1 μF should also be easy to obtain. Making $R$ the subject of the formula and substituting for $C = 1$ μF gives:

$$p.r.f. = \frac{1.44}{C(R_1 + 2R_2)}$$

$$R = \frac{480 \times 10^3}{100} = 4.8 \times 10^3 = 4.8 \text{ kΩ}$$

**Example 12.4**

Design a 5 V 50 Hz square wave generator using a 555 timer.
The 555 timer

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The complete circuit of the 5 V 50 Hz square wave generator is shown in Fig. 12.10.

A variable pulse generator

Fig. 12.11 shows how a variable pulse generator can be constructed using two 555 times (or one 556 dual timer). The first timer, IC1, operates in astable mode while the second timer, IC2, operates in monostable mode. The

Solution

Using the circuit shown in Fig. 12.7, when \( R_2 \gg R_1 \), the value of p.r.f. can be calculated from:

\[
p.r.f. = \frac{0.72}{CR_2}
\]

We shall use the minimum recommended value for \( R_1 \) (i.e. 10 kΩ) and ensure that the value of \( R_2 \) that we calculate from the formula is at least ten times larger in order to satisfy the criteria that \( R_2 \) should be very much larger than \( R_1 \). When selecting the value for \( C \) we need to choose a value that will keep the value of \( R_2 \) relatively large. A value of 100 nF should be about right and should also be easy to locate. Making \( R_1 \) the subject of the formula and substituting for \( C = 100 \text{ nF} \) gives:

\[
R_2 = \frac{0.72 \times C}{p.r.f. \times 50 \times 100 \times 10^{-9}} = \frac{0.72 \times 100 \times 10^{-6}}{5 	imes 10^{-9}} = 144 \times 10^6 = 144 \text{ kΩ}
\]

Alternatively, the chart shown in Fig. 12.9 can be used.

The value of \( R_2 \) is thus more than 100 times larger than the value that we are using for \( R_1 \). As a consequence the timer should produce a good square wave output.

Figure 12.10 A 5 V 50 Hz square wave generator (see Example 12.4)

Figure 12.11 A variable pulse generator using two 555 timers
12 The 555 timer

p.r.f. generated by IC1 is adjustable by means of switch selected capacitors, \( C_1 \) to \( C_3 \), together with variable resistor, VR. The output from IC1 (pin 3) is fed via \( C_6 \) to the trigger input of IC2 (pin 2).

The monostable period of IC2 is adjustable by means of switch selected capacitors, \( C_6 \) to \( C_8 \), together with variable resistor, VR. The output from IC2 (pin 3) is fed to the output via VR.

The p.r.f. is adjustable over the range 10 Hz to 10 kHz while pulse widths can be varied from 50 \( \mu \)s to 50 ms. The output voltage is adjustable from 0 V to 10 V. Finally, \( R_5 \) is included in order to limit the output current and provide a measure of protection in the event of a short-circuit present at the output.

**Practical investigation**

**Objective**

To investigate the operation of a 555 monostable timer circuit.

** Simulator**

Breadboard, 5 V d.c. power supply, 555 timer, resistors of 10 k\( \Omega \) (two required), 220 \( \Omega \), 100 k\( \Omega \) and 1 M\( \Omega \) 5% 0.25 W, capacitors of 10 \( \mu \)F and 100 \( \mu \)F 16 V, LED, two normally open (NO) pushbutton switches, stopwatch or wristwatch with seconds display.

**Procedure**

Connect the circuit as shown in Fig. 12.12 with \( C = 10 \mu \)F and \( R = 100 \, k\Omega \). Connect the supply and press the 'stop' button. The LED should be off (indicating that the output is at 0 V).

Observe the time display and, at a convenient point, press the 'start' button. The LED should become illuminated after a period of about 1 s (this will probably be too short an interval to be measured accurately). Record the monostable time period (i.e. the time between pressing the 'start' button and the LED becoming illuminated) (see Table 12.2).

Repeat the procedure for each of the remaining \( C-R \) values shown in Table 12.2. Note that you can interrupt the timing period at any time by pressing the 'stop' button.

**Calculations and graph**

Record your results in Table 12.2. For each pair of\( C-R \) values calculate the product of \( C \) (in \( \mu \)F) and \( R \) (in M\( \Omega \)). Plot a graph showing corresponding values of monostable time plotted against corresponding values of \( C \times R \) using the graph layout shown in Fig. 12.13.

**Conclusions**

Comment on the shape of the graph. Is this what you would expect? Measure the slope of the graph and use this to confirm the relationship for the monostable timing period quoted on page 225. If the graph is not linear can you suggest any reasons for this?

**Table 12.2** Table of results for the monostable timer circuit

<table>
<thead>
<tr>
<th>( C ) ( \mu )F</th>
<th>( R ) ( k\Omega )</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>470</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>470</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1 M</td>
<td></td>
</tr>
</tbody>
</table>
12 The 555 timer

Further work

Connect a digital multimeter on the 20 V d.c. range so that you can accurately measure the d.c. voltage that appears between pin 6 (threshold input) and 0 V. With $C = 100 \mu F$ and $R = 1 \text{ M} \Omega$ press the ‘start’ button and then measure the voltage at pin 6 at intervals of 10 s over the range 0 to 120 s. Particularly note the voltage reached at the end of the monostable timing period (this should be exactly two-thirds of the supply voltage). Plot a graph of voltage against time and justify the shape of this graph.

Important formulae introduced in this chapter

Monostable 555 timer:
(page 225)

\[ t_{\text{on}} = 1.1 \, CR \]

Astable 555 timer:
(page 227)

\[ t_{\text{on}} = 0.693 \, C(R_1 + R_2) \]
\[ t_{\text{off}} = 0.693 \, CR_2 \]
\[ t = 0.693 \, C(R_1 + 2R_2) \]

When $R_2 > > R_1$: (page 228)

\[ p.r.f. = \frac{0.72}{CR_2} \]
\[ \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} = 50\% \]

Problems

12.1 Design a timer circuit that will produce a 10 V 2 ms pulse when a 10 V negative-going trigger pulse is applied to it.

12.2 Design a timer circuit that will produce time periods that can be varied from 1 s to 10 s. The timer circuit is to produce a +12 V output.

12.3 Design a timer circuit that will produce a 67% duty cycle output at 400 Hz.

12.4 Design a timer circuit that will produce a square wave output at 1 kHz.

12.5 Refer to the variable pulse generator circuit shown in Fig. 12.11. Identify the component(s) that:

(a) provides variable adjustment of pulse width
(b) provides decade range selection of pulse width
(c) limits the range of variable adjustment of pulse width
(d) provides variable adjustment of p.r.f.
(e) provides decade range selection of p.r.f.
(f) limits the range of variable adjustment of p.r.f.
(g) provides variable adjustment of output amplitude
(h) protects IC2 against a short-circuit connected at the output